ECON 1110: Intermediate Macro

Lecture 6 Supplement

1 Consumption and Savings

Here we are looking a two period model. The consumer has income y today and y' in the next period. Similarly, taxes are t today and t' in the next period. In order to shift consumption between the two periods, the consumer can save or borrow at a common rate r. This choice is represented by the variable s, which is positive in the case of savings and negative in the case of borrowing.

Consumption today and tomorrow are represented by c and c'. The budget constraints for the two periods are then

$$\begin{aligned} c + s &= y - t \\ c' &= y' - t' + (1+r)s \end{aligned}$$

We can eliminate s to combine these two into a single present value budget constraint.

$$\begin{aligned} &\Rightarrow s = y - t - c \\ &\Rightarrow c' = y' - t' + (1+r)(y - t - c) \\ &\Rightarrow (1+r)c + c' = (1+r)(y-t) + y' - t' \\ &\Rightarrow c + \frac{c'}{1+r} = y - t + \frac{y' - t'}{1+r} \equiv we \end{aligned}$$

The last term is what we'll call wealth, since it is the present value of the income stream. It determines the size of the budget set, while $1+r$ determines the slope. Two consumers with the same wealth will make the same consumption-savings choices, even if y and y' differ.

Now let's solve for the consumer's optimal choice. In general a consumer's preferences are represented by

$$U(c, c') = u(c) + \beta u(c')$$

for some per-period utility function u and discount rate $\beta \in (0, 1)$. Usually, we will look at the case where $u(c) = \log(c)$, though as we saw in assignment 4, there are other options. Then we have

$$U(c, c') = \log(c) + \beta \log(c')$$

We can use the budget constraints to formulate the consumer's problem as purely one of choosing the optimal s

$$\max_s \ \log(y - t - s) + \beta \log(y' - t' + (1 + r)s)$$

To find this, we simply take the derivative with respect to s

$$\begin{aligned}
&\frac{-1}{y - t - s} + \frac{\beta(1 + r)}{y' - t' + (1 + r)s} = 0 \\
\Rightarrow\ &\frac{1}{y - t - s} = \frac{\beta(1 + r)}{y' - t' + (1 + r)s} \\
\Rightarrow\ &y' - t' + (1 + r)s = \beta(1 + r)(y - t - s) \\
\Rightarrow\ &\beta(1 + r)s + (1 + r)s = \beta(1 + r)(y - t) - (y' - t') \\
\Rightarrow\ &(1 + \beta)(1 + r)s = \beta(1 + r)(y - t) - (y' - t') \\
\Rightarrow\ &s^* = \frac{\beta(1 + r)(y - t) - (y' - t')}{(1 + \beta)(1 + r)}
\end{aligned}$$

We can also use the second line of the above derivation to see that

$$\begin{aligned}
\Rightarrow\ &\frac{y' - t' + (1 + r)s}{y - t - s} = \beta(1 + r) \\
\Rightarrow\ &\frac{c'}{c} = \beta(1 + r)
\end{aligned}$$

Thus we can see that if $\beta(1 + r) = 1$, then there will be perfect consumption smoothing,

$c' = c.$