Econ 3070: Midterm

There are two questions on the exam, each with four parts. Each part is worth 12 points, and you get 4 points just for writing your name down. You have 80 minutes to complete the exam. Please show and hand in all your work. Good luck!

1 Be At One With Nature?

Consider a Malthusian model in which there is a depletable natural resource (climate, flora, fauna, etc.), which we denote by H. Production is undertaken using both the stock of this resource and labor as factors, as in

$$Y = zH^{\alpha}L^{1-\alpha}$$

This resource naturally regenerates at rate m, but humans also deplete it in proportion to their per capita output level, so that

$$g_H = \frac{\dot{H}}{H} = m - \delta y$$

In conjunction, we also have classical Malthusian population dynamics given by

$$g_L = \frac{\dot{L}}{L} = \theta(y - \bar{y})$$

where $y = \frac{Y}{L}$ is the per capita output level.

(a) Express output per capita y as a function of the per capita resource level $h = \frac{H}{L}$. Using this, find the law of motion for the per capita resource level $\dot{h}$ as functions of only h and model parameters.

(b) Draw a plot of $\dot{h}$ as a function of h itself. Next, draw a plot of the positive and negative forces acting on h, as we do with the Solow model.

(c) Solve for the steady state resource level h^* and use this to find the steady state output per capita y^*.

(d) In steady state, what are the growth rates g_H and g_L? Under what conditions are these positive?

2 Stack It Up (Solow's Version)

Consider a Solow model in which we have a staggered production setup as in modern supply chains. There is an intermediate type of capital K_1 that is used only for the production of final capital K_2. The final good production function is then

$$Y = K_2^{\alpha} L_2^{1-\alpha}$$

while the laws of motion for the different types of capital are given by

$$\begin{aligned} \dot{K}_1 &= sY - \delta K_1 \\ \dot{K}_2 &= K_1^{\alpha} L_1^{1-\alpha} - \delta K_2 \end{aligned}$$

The labor allocation should satisfy $L_1 + L_2 = L$, where L is the total population. There is no population growth, so L is constant.

(a) Reformulate the production function and the laws of motion for capital in per capita terms. Use the notation $k_i = \frac{K_i}{L}$ and $\ell_i = \frac{L_i}{L}$. Here $\ell_i \in [0, 1]$ is the labor fraction for capital type i.

(b) Draw a phase diagram in (k_1, k_2) space. Show the lines where $\dot{k}_1 = 0$ and $\dot{k}_2 = 0$, draw arrows indicating the direction of flow in each region, and label their intersection as the steady state (k_1^*, k_2^*).

(c) Solve for the steady state values k_1^* and k_2^* algebraically. Then use these to find an expression for the steady state per capita output y^*.

(d) Find the value of ℓ_1 that maximizes steady state output. Similarly, find the savings rate that maximizes steady state consumption $c^* = (1-s)y^*$. [**Hint**: Take logs *before* taking the derivative when maximizing.]