Econ 3070: Midterm Exam

Question 1: Malthus + Solow

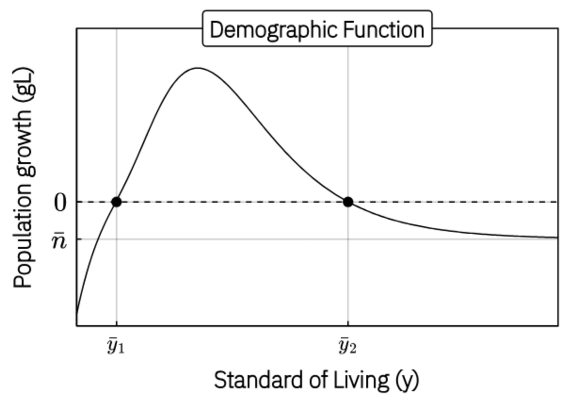
We're combining Malthus and Solow into a Frankenmodel. At any given time, we have a population L and a capital stock K. Production is Cobb-Douglas with constant technology, so that $Y = zK^{\alpha}L^{1-\alpha}$. Capital evolves according to the usual equation

$$\dot{K} = sY - \delta K$$

Meanwhile, population evolves according to the Malthusian demographic law

$$\frac{\dot{L}}{L} = n(y)$$

where $y = Y/L$ is the standard of living. The function $n(\cdot)$ is of the "Kremer" variety but we assume that it eventually goes negative, that is $\lim_{y\to\infty} n(y) = \bar{n} < 0$. And in particular there will be two points $\bar{y}_1$ and $\bar{y}_2$ for which $n(\bar{y}_1) = n(\bar{y}_2) = 0$, as in



(a) Use the production function to derive an expression for the standard of living y as a function of technology z and the capital to labor ratio $k = K/L$.

(b) Express the growth rates of capital g_K and population g_L purely as functions of y. Using this, find the growth rate of capital per worker g_k as a function of y and then do the same for growth in the standard of living g_y.

(c) Plot g_K and g_L as functions on y on the same graph. Their intersection should constitute a steady state with $g_y = g_k = 0$. Supposing there is some steady state population growth rate n^*, what is the resulting standard of living y^*?

(d) What are the possible long-run outcomes in this case? Is there population growth, stagnation, or collapse? How does this depend on model parameters?

Question 2: Plumpton's Revenge

Consider a Ramsey style model in which there two distinct communes $i \in \{1, 2\}$. They are of constant and equal size ($L_i = 1$). They have the same flow utility function over per capita consumption c_i given by

$$u_i(c_i) = \log(c_i)$$

They differ only in their rate of time discounting ρ_i. Without loss of generality, assume that $\rho_1 < \rho_2$, meaning commune 1 is more patient. Production is undertaken jointly by all commune workers ($L = L_1 + L_2$) according to the function

$$F(K, L) = K^\alpha L^{1-\alpha}$$

Finance is also handled jointly, meaning both communes earn a rate of return r on assets A_i, and the market clearing condition for assets is $A_1 + A_2 = K$.

(a) Write down the per capita budget constraint of commune i. Using this, formulate the Hamiltonian and characterize the optimal path of consumption $c_i = C_i/L_i$ and assets $a_i = A_i/L_i$ taking factor prices r and w as given.

(b) We wish to study how inequality evolves in this setting. Let aggregate consumption be $c = c_1 + c_2$ and the consumption shares be $\tilde{c}_i = c_i/c$. Derive a law of motion for $\tilde{c}_i$ and find their limiting (steady state) values.

(c) Find the steady state interest rate r and capital level $k = K/L$. How do these compare to those in the standard Ramsey model? Describe and plot what the paths of consumption c_i and assets a_i would look like in this model.

(d) Suppose the government wishes to limit inequality between the communes. Propose and fully characterize a budget neutral taxation policy that would ensure eventual equality in consumption ($c_1 = c_2$). [**Hint**: consider using an asset tax wherein a commune with assets a_i pays a tax/subsidy of $\tau_i a_i$.]