Econ 3070: Midterm Exam

There are two questions on the exam, each with four parts. Each part is worth 12 points, and you get 4 points just for writing your name down.

You have 3 hours to complete the exam. Be thorough and show all your work. Feel free to consult your notes, lecture notes, recitation notes, and the textbook during the exam. Submit your completed exam to Canvas when you're done, at the latest by **March 23rd, 10:30am EDT**. Good luck!

1 Rise of the Machines

Consider a variant of the Solow model in which there are two types of capital: traditional capital K and robotic capital M, which is a perfect substitute for labor L. The production function is given by

$$Y = K^{\alpha}(L + M)^{1-\alpha}$$

Labor grows at rate n, and the laws of motion for the two types of capital are

$$\dot{K} = s_k Y - \delta K$$
$$\dot{M} = s_m Y - \delta M$$

Meanwhile, consumption is $C = (1 - s_k - s_m)Y$.

(a) Reformulate the model in per capita terms. In particular, express $\dot{k}$ and $\dot{m}$ in terms of only k and m (and parameters)?

(b) Now reformulate the model in terms of the (hopefully) stationary variables

$$a = \frac{k}{m} \quad \text{and} \quad b = \frac{m}{1+m}$$

That is, derive $\dot{a}$ and $\dot{b}$ in terms of only a and b (and parameters).

(c) Draw a phase diagram for the joint dynamics of a and b. Specifically, draw the nullclines where $\dot{a} = 0$ and $\dot{b} = 0$, draw arrows indicating the resulting direction of flow in each region, and draw some sample paths.

(d) What are the possible steady states of this system? Which ones are stable? [**Hint**: think about the different assumptions you can make in drawing the phase diagram in (c).]

2 Leisure Class

Consider a Ramsey-type model in which utility over consumption and disutility of working are additively separable, so that

$$U = \int_0^\infty \left[u(c(t)) - v(h(t))\right] L(t) \exp(-\rho t) dt$$

where u is concave and v is convex. Labor grows at constant rate n. The production function combines capital K and labor according to

$$Y = K^\alpha (AH)^{1-\alpha}$$

where $H = hL$ is total hours worked and workers are paid a wage w per unit time. Technology A grows at rate g and capital depreciates at rate δ.

(a) Express the budget constraint and production function for this problem in per capita terms.

(b) Find the factor prices w and R (for H and K, respectively) in this model and express them in per capita terms.

(c) Write down the Hamiltonian for the consumer optimization problem and derive conditions for the dynamic evolution of c, h, and k.

(d) Now suppose the utility components are given by

$$u(c) = \frac{c^{1-\theta} - 1}{1 - \theta} \qquad \text{and} \qquad v(h) = \frac{h^{1+\eta}}{1+\eta}$$

What is the limiting behavior for c, h, and k in terms of growth rates?