

ECON 2130: Macro 2.0

Final Exam

Due: Tuesday April 21st, 10:00am on Blackboard

For both problems below, there is assumed to be a consumer with log utility, so that the Euler equation is simply $r = \rho + g_c$.

1 Decreasing Returns

Consider an expanding varieties model in which there are decreasing returns to the production of intermediate goods. In particular, each good is produced with labor according to

$$y_i = \ell_i^{1-\alpha}$$

and these goods are linearly aggregated into total output with

$$Y = \int_0^N y_i di$$

There is a unit mass of labor divided between production labor $P = \int_0^N \ell_i di$ and research labor R , meaning $1 = P + R$. Finally, the research production function is

$$\dot{N} = \gamma NR$$

(a) Solve the static production problem in this setting taking N and P as given. In particular, find the quantity produced by the intermediate firm y_i , the resulting profits π_i , total output Y , and the wage w .

(b) Now use the free entry condition to solve for the research labor allocation R and the resulting growth rate g . Under what conditions is there positive growth?

(c) Solve the social planner's problem in this setting. What can be said about the efficiency of the decentralized equilibrium level of research and growth?

2 Targeted Innovation

Consider a model of creative destruction with the final good aggregator

$$\log(Y) = \int_0^1 \alpha_i \log(y_i) di$$

where $\alpha_i > 0$ and $\int_0^1 \alpha_i di = 1$. As usual, intermediate goods are produced linearly using labor according to

$$y_i = q_i \ell_i$$

where q_i is the productivity of the firm producing good i . Total production labor satisfies $P = \int_0^1 \ell_i di$.

Entrants can do research that is **directed to specific intermediate goods**, which increments the productivity of that good by a factor $1 + \lambda$. Let the innovation rate for good i be

$$\tau_i = \gamma R_i$$

where R_i is the research directed at good i . Let the aggregates be $\tau = \int_0^1 \tau_i di$ and $R = \int_0^1 R_i di$. Finally there is a unit mass of labor so that $1 = P + R$.

(a) Solve the static production problem for this model. Report the values for profit π_i and wage w taking total output Y and production labor P as given.

(b) Use the free entry condition to find the dynamic outcome. Specifically, report values for total research R and the innovation rates τ_i .

(c) Define an appropriate aggregate productivity index Q and find an expression for the overall growth rate of total output Y . You can assume that the distribution of α_i is reasonable and that $\int_0^1 \alpha_i^2 di = \sigma$.