Problem Set 3

Please submit your solutions before recitation on Thursday, April 22nd. You are encouraged to work together on these assignments, but submit only your own work.

For both problems assume consumers have logarithmic utility with discount rate ρ and that there is no population growth.

1 Targeted Innovation

Consider a model of creative destruction with the final good aggregator

$$\log(y) = \int_0^1 \alpha_i \log(y_i)\ di$$

where $\alpha_i > 0$ and $\int_0^1 \alpha_i\ di = 1$. Intermediate goods are produced linearly using labor according to

$$y_i = q_i \ell_i$$

where q_i is the productivity of the firm producing good i. Total production labor satisfies $P = \int_0^1 \ell_i\ di$.

Entrants can do research that is **directed to specific intermediate goods**, which increments the productivity of that good by a factor $1 + \lambda$. Let the innovation rate for good i be

$$\tau_i = \gamma R_i$$

where R_i is the research directed at good i. Let the aggregates be $\tau = \int_0^1 \tau_i\ di$ and $R = \int_0^1 R_i\ di$. Finally there is a unit mass of labor so that $1 = P + R$.

(a) Solve the static production problem for this model. Report the values for profit π_i and wage w taking total output y and production labor P as given.

(b) Use the free entry condition to find the dynamic outcome. Specifically, report values for total research R and the innovation rates τ_i.

(c) Define an appropriate aggregate productivity index Q and find an expression for the overall growth rate of total output y. You can assume that the distribution of α_i is reasonable and that $\int_0^1 \alpha_i^2\ di = \sigma$.

Patents and Innovation

Consider a model of creative destruction with the final good aggregator

$$\log(y) = \int_0^1 \alpha_i \log(y_i)\ di$$

Intermediate goods are produced linearly using labor according to

$$y_i = q_i \ell_i$$

where q_i is the productivity of the firm producing good i. Total production labor satisfies $P = \int_0^1 \ell_i\ di$.

Entrants can do research which increments the productivity of that good by a factor $1 + \lambda$. Let the innovation rate for good i be

$$\tau = \gamma R$$

where R is the amount of research labor used, and there is a unit mass of labor so that $1 = P + R$. However, **patents expire** at Poisson rate b, in which case firms lose their monopoly and the good is produced competitively.

(a) Solve the static production problem for this model. Report the values for profit π_i and labor ℓ_i taking total output y and production labor P as given.

(d) Let the fraction of monopoly product lines be μ. Assuming fixed τ, write down the evolution equation for μ and use this to find the steady state value for μ?

(c) Find the steady state levels of output y and growth rate g in this economy. How do these depend on the patent expiry rate?

(d) Note that the social planner's problem is the same as that solved in lecture. How would we use these results to find the optimal value for b and what are the tradeoffs involved?