Problem Set 2

Please submit your solutions before recitation on Friday, March 5th. You are encouraged to work together on these assignments, but submit only your own work.

1 Labor and Leisure

Consider the Ramsey model with population growth at rate n and labor-augmenting technology growth g. Suppose the commune derives utility not only from consumption, but also from leisure ℓ. The lifetime utility of the commune is given by

$$U = \int_0^\infty u(c(t), \ell(t)) \exp(-\rho t) L(t) dt$$

$$u(c, \ell) = \frac{\left[c^\eta \ell^{1-\eta}\right]^{1-\theta} - 1}{1-\theta}$$

where c is consumption per person, $\ell \in [0,1]$ is leisure time, $0 < \eta < 1$, and $\theta > 0$. The production technology of the representative firm in this economy is described by the following Cobb-Douglas production function

$$Y = K^\alpha (AhL)^{1-\alpha}$$

where $h = 1 - \ell$ is working time. Assume capital depreciates at rate $\delta > 0$. Note that workers are paid a wage w *per unit time.*

(a) Characterize the balanced growth path (BGP) of this model. In particular, derive the growth rates of the different variables g_K, g_I, g_C, g_Y, g_c and g_h as functions of the exogenous parameters of the model.

(b) Reformulate the model in stationary terms and find the dynamic equilibrium. In solving the commune's problem, notice that now you need to derive the optimal working time h. Comment on the intuition for this optimal labor supply.

(c) Solve for the steady state of the model. In addition, how would you proceed showing whether the steady state of this model is stable, unstable, or saddle-path stable? Just describe the steps you would take and what criteria you would use to show stability.

(d) Suppose now that the momentary utility function is changed to

$$u(c, \ell) = \log(c - \psi(1-\ell)^\gamma)$$

Repeat the above steps (except for the stability part) for this new environment.

2 Workers and Capitalists

Suppose that we have a standard Ramsey model with no population growth ($n = 0$) and no technological growth ($g = 0$). For convenience, let $A(t) = 1$ and $L(t) = L$. Utility is logarithmic over consumption with discount rate ρ and the production function is Cobb-Douglas.

However, there are two distinct classes in this society, workers (w) and capitalists (k), with populations

$$L_w = (1-\beta)L \qquad \text{and} \qquad L_k = \beta L$$

All capital is owned by capitalists, who make investment decisions and do not work or receive wages. Workers own no capital but do work and receive wages.

(a) First, reformulate the model using the following change of variables

$$\begin{aligned} a &= \frac{A}{\beta L} & k &= \frac{K}{(1-\beta)L} \\ c_k &= \frac{C_k}{\beta L} & c_\ell &= \frac{C_\ell}{(1-\beta)L} \end{aligned}$$

and solve for the capitalists optimal investment decisions. You should end up with equations describing the evolution of k, c_k, and c_ℓ. [*Hint*: be careful with factor prices.]

(b) Find the steady state of values for k, c_k, and c_ℓ in this model. How do these change with the capitalist share β?

(c) Characterize the stability of the steady state and determine how the speed of convergence is affected by β.

(d) To analyze inequality, calculate the Gini coefficient for this economy and determine how it depends on β.