Economics 101

Lecture 3 - Consumer Demand

1 Intro

First, a note on wealth and endowment. Varian generally uses wealth (m) instead of endowment. Ultimately, these two are equivalent. Given prices p, if we have endowment e, our wealth is simply $w = p \cdot e$. Conversely, if we have wealth w, this is the same as have an endowment e given by

$$e_i = \frac{w}{\sum_j p_j}$$

so that $p \cdot e = w$. Note that there are many other endowments that lead to wealth w.

Last lecture we investigated how consumers make choices by maximizing their utility subject to the budget constraint. Given prices and endowment, we found what the consumer's optimal choice was. Now we'll formalize this

Demand Function: For any prices and endowment, a demand function gives the bundle of goods $x(p, e)$ optimally chosen by the consumer.

Notice that give the relationship between endowment and wealth, it is equivalent to define demand simply as a function of prices and wealth. Sometimes, to avoid confusion I will add superscripts to denote which function I'm talking about. These functions are related by

$$x^e(p, e) = x^w(p, p \cdot e)$$

We are free to do this because the only thing that determines the shape of the budget set is normalized prices and wealth. Changing the endowment while keeping wealth constant does not change the budget line. It only changes where the endowment lies on the budget line, which does not affect the optimal choice.

Also note that we assume throughout that x is a function, meaning that the is one and only one optimal choice. In general, it may be there are multiple choice of x that yield the same maximal utility, in which case x would be a set-valued function. Assuming that utility is strictly increasing rids us of this problem.

2 Comparative Statics

Often we will be interested in how a consumer's choice will vary with certain economic variables such as market prices, wealth, or certain preference parameters. We'll be interested primarily in two of these.

Price Effects: Holding wealth constant, how does price affect the optimal choice?

$$\frac{\partial x_i(p,w)}{\partial p_i} \gtrless 0$$

Wealth Effects: Holding prices constant, how does wealth affect the optimal choice?

$$\frac{\partial x_i(p,w)}{\partial w} \gtrless 0$$

We'll look at each of these separately.

2.1 Wealth Effects

Increasing wealth while holding prices constant simply shifts the budget line outwards while keeping the slope the same. Decreasing wealth does the opposite.

Each change in wealth will lead to a new optimal consumption bundle. Fixing a certain price vector, if we were to map out the choice for each possible value of w, this would sketch out what is known as the wealth offer curve. This would necessarily start from the origin and move out from there.

Proposition 1. *If $w' \geq w$, then $x(p,w') \not< x(p,w)$.*

Since the budget set becomes strictly larger when wealth increases to w', if it optimal to decrease consumption in every good upon this increase in wealth, then $x(p, w')$ should have been the optimal choice for (p, w) as well. Notice with vectors, $\not\leq$ is not equivalent to $\geq$.

With this, we can now define two classes of goods, depending on how their demand responds to changes in wealth.

Normal Good: Consumption is increasing in wealth

$$\frac{\partial x_i(p, w)}{\partial w} \geq 0$$

Inferior Good: Consumption is decreasing in wealth

$$\frac{\partial x_i(p, w)}{\partial w} \leq 0$$

We'll often say that these are properties of goods. Really, they are properties of utility functions (and their associated demand functions). Additionally, this is a local property, so some goods may be normal for certain prices or wealth levels and inferior for others. Often, using these terms implicitly means that the above conditions hold for all (p, w). An immediate implication of the proposition above is that at any (p, w), there must be at least one normal good.

We may also be interested in how the fraction of income spent on a particular good changes with wealth. The fraction of wealth spent on good i is

$$\frac{p_i x_i(p, w)}{\sum_j p_j x_j(p, w)} = \frac{p_i x_i(p, w)}{w}$$

If this fraction is increasing, then the good is called a <u>luxury good</u>. Otherwise, it is called a <u>necessary good</u>. When this fraction is constant for all w, we say that preferences are <u>homothetic</u>.

If the optimal choice can be written in the form

$$x(p, w) = z(p) \cdot w$$

then

$$\frac{p_i x_i(p, w)}{\sum_j p_j x_j(p, w)} = \frac{p_i z_i(p)}{\sum_j p_j z_j(p)}$$

and thus the fraction is independent of w, meaning preferences are homothetic. Notice that this is the case for Cobb-Douglas, CES, and Leontieff preferences. It is not the case for the modified Cobb-Douglas preferences we looked at.

Example 1. Consider the utility function

$$u(x_1, x_2) = x_1^{\gamma_1} + x_2^{\gamma_2}$$

where $\gamma_1 = \frac{3}{4}$ and $\gamma_2 = \frac{1}{4}$. Let $p_1 = p_2 = 1$. Find the optimal choices as functions of w. Which good is inferior and which is normal? What about luxury vs. necessary?

2.2 Price Effects

Now we're going to do the same exercise but for price changes instead of wealth changes. Again, here we change prices while keeping wealth fixed. Decreasing the price of good i will rotate the budget set out towards good i. Increasing the price of good i will do the opposite. We have two analogous classifications for price effects

Ordinary Good: Consumption decreases when price increases

$$\frac{\partial x_i(p, w)}{\partial p_i} \leq 0$$

Giffen Good: Consumption increases when price increases

$$\frac{\partial x_i(p, w)}{\partial p_i} \geq 0$$

As before, these are local properties and may or may not hold at different prices and wealth levels. Giffen goods are remarkably hard to spot in the wild.

In addition to direct price effects, there are also cross-good effects. That is, we can look at how the change in the price of good i affects consumption of good j. We classify these effects as

Substitutes: Consumption of good i increases with the price of good j

$$\frac{\partial x_i(p, w)}{\partial p_j} > 0$$

Complements: Consumption of good i decreases with the price of good j

$$\frac{\partial x_i(p,w)}{\partial p_j} < 0$$

One important things to note is that in settings where there are only 2 goods, it can be that good i is a substitute for good j, but good j is not a substitute for good i.

3 Price Effects

Up until now, we have been considering changes in prices or wealth keeping the other fixed. Now we wish to determine the total of effect of a price change on consumption. To do this, we must switch back to thinking about endowments. This way, price has a direct effect as well as a wealth effect, since changing prices also changes the value of a consumers endowment, that is, their wealth.

To avoid confusion, I will explicitly label the functions $x^e(p,e)$ and $x^w(p,w)$. Recall that these two functions are related by

$$x^e(p,e) = x^w(p, p \cdot e)$$

Differentiating the above equation with respect to p_i, we arrive at

$$\underbrace{\frac{\partial x_i^e(p,e)}{\partial p_i}}_{\text{net price effect}} = \underbrace{\frac{\partial x_i^w(p,w)}{\partial p_i}}_{\text{pure price effect}} + \underbrace{\frac{\partial x_i^w(p,w)}{\partial w}}_{\text{pure wealth effect}} \times e_i$$

This decomposes the net price effect into the pure price effect and the wealth effect. Rearranging yields

$$\frac{\partial x_i^w(p,w)}{\partial p_i} = \frac{\partial x_i^e(p,e)}{\partial p_i} - \frac{\partial x_i^w(p,w)}{\partial w} \times e_i$$

The value of the endowment e here is arbitrary. Suppose it is the case that $e = x^e(p,e) = x^\star$. Assuming increasing utility, this will satisfy $w = p \cdot e = p \cdot x^\star$. Therefore

$$\frac{\partial x_i^w(p,w)}{\partial p_i} = \frac{\partial x_i^e(p,x^\star)}{\partial p_i} - \frac{\partial x_i^w(p,w)}{\partial w} \times x_i^\star$$

The above equations is called the Slutsky equation. We can plug this equation into the decomposition equation above to yield an alternate form

$$\frac{\partial x_i^e(p,e)}{\partial p_i} = \frac{\partial x_i^e(p,x^\star)}{\partial p_i} - \frac{\partial x_i^w(p,e)}{\partial w} \times (x_i - e_i)$$

Now consider the following result.

Lemma 1. *When the optimal choice is the endowment, the net substitution effect is negative. That is, when $x^e(p,e) = e = x^\star$, we have*

$$\frac{\partial x_i^e(p,x^\star)}{\partial p_i} \leq 0$$

Proof. Consider if we go from p_i to p_i'. Let the new price vector with other elements unchanged to p'. We wish to show that

$$\frac{x_i^e(p',x^\star) - x_i^e(p,x^\star)}{p_i' - p_i} \leq 0$$

Since $x^\star$ is the optimal choice in $B(p,x^\star)$, it is preferred to every point in $B(p,e)$. If $x^e(p',x^\star) = x^e(p,x^\star)$, then we are all set. Otherwise, the new point $x' = x^e(p',x^\star)$ must be strictly preferred to $x^\star$. In this case, x' cannot be in $B(p,x^\star)$. It is however, in $B(p',x^\star)$. These imply

$$p \cdot x' > p \cdot x^\star \qquad \text{and} \qquad p' \cdot x' \leq p' \cdot x^\star$$

Rearranging

$$-p \cdot (x' - x^\star) < 0 \qquad \text{and} \qquad p' \cdot (x' - x^\star) \leq 0$$

Summing

$$(p' - p) \cdot (x' - x^\star) < 0$$

and since p' and p differ only in the i^{th} dimension

$$(p_i' - p_i)(x_i' - x_i^\star) < 0$$
$$\Rightarrow \frac{x_i' - x_i^\star}{p_i' - p_i} < 0$$

when $p_i' \neq p_i$, as we wished to show. Taking the limit as $p_i' \to p_i$, yields the derivative form. □

Tying this back to the Slutsky equation, we can also see the following result

Proposition 2. *If good i is a normal good, then it is also an ordinary good.*

Proof. Being a normal good means that

$$\frac{\partial x_i^w(p,w)}{\partial w} \geq 0$$

Looking back to the Slutsky equation, having established that the first term is negative and subtracting a positive term, we conclude that

$$\frac{\partial x_i^w(p,w)}{\partial p_i} \leq 0$$

meaning good i is an ordinary good. □

Using the alternate form of the Slutsky equation, we can conclude using a proof similar to the above that

Proposition 3. *If good i is a normal good and $x_i \geq e_i$, then demand is decreasing in the price of i, that is*

$$\frac{\partial x_i^e(p,e)}{\partial p_i} \leq 0$$

Example 2 (Cobb-Douglas)**.** Recall from last lecture that with Cobb-Douglas utility, the agent spends a fraction α_i of his wealth on good i. This implies demand functions of the form

$$x_i^e(p,e) = \frac{\alpha_i(p \cdot e)}{p_i} \qquad \text{and} \qquad x_i^w(p,w) = \frac{\alpha_i w}{p_i}$$

All goods are normal goods with C-D since

$$\frac{\partial x_i^w(p,w)}{\partial w} = -\frac{\alpha_i}{p_i} > 0$$

Thus they are ordinary goods as well, as we can check

$$\frac{\partial x_i^w(p,w)}{\partial p_i} = -\frac{\alpha_i w}{p_i^2} < 0$$

Now let's find the net price effect

$$\frac{\partial x_i^e(p,e)}{\partial p_i} = \frac{p_i \alpha_i e_i - \alpha_i(p \cdot e)}{p_i^2} = \frac{\alpha_i}{p_i^2}\Big[p_i e_i - w\Big] < 0$$

So here the net price effect is negative, regardless of whether x_i is larger or smaller than e_i. This is fine, since that was only a sufficient condition.

4 Offer Curves

Recall before we discussed the concept of a wealth offer curve. Given any price p, this function mapped from any price level w into the optimal choice $x^w(p, w)$. Now we'll consider the same concept for prices. Thus, the price offer curve is a function that maps from prices p into an optimal choice $x^w(p, w)$, given some w or e.

Consider the case of only two goods. Here, we need know only the price ratio, rather than each price individually. So the offer curve is a function that maps from one positive real number (the price ratio) into an allocation $x \in \mathbb{R}^2_+$, meaning it is a curve in 2-dimensions.

Imagine we broke the Cartesian plane into 4 quadrants as usual (1 in the northeast, increasing counterclockwise), but we center them at the endowment rather than the origin. We can say the following about the offer curve

1. The offer curve can never lie quadrant 1. If a consumption bundle has more of both good 1 and 2 than the endowment, it must cost more and thus be outside of the budget set.

2. So long as utility is increasing, the optimal choice will always lie on the budget line, so the offer curve cannot lie in quadrant 3.

3. As $p_1 \to 0$, $x_1 \to \infty$. Similarly, as $p_2 \to 0$, $x_2 \to \infty$. If the price of a good goes to zero, then the consumer should start optimally choosing more and more of it.

4. The offer curve is continuous. We didn't prove it formally, but continuity of the optimal choice arises from concavity of the utility function. This is result is called Berge's Maximum Theorem.

5. Since the offer curve is continuous and must pass between quadrants 2 and 4 without going through quadrants 1 and 3, it must pass through the endowment at some point.

Offers curves can be useful for understanding the intuition behind certain results. They'll be of use next lecture when we start discussing Walrasian equilibrium.

5 Labor Supply