Back to Basics: Basic Research Spillovers, Innovation Policy and Growth*

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Abstract

This paper introduces a general equilibrium model of endogenous technical change through basic and applied research. Basic research differs from applied research in the nature and the magnitude of the generated spillovers. We propose a novel way of empirically identifying these spillovers and embed them in a framework with private firms and a public research sector. After characterizing the equilibrium, we estimate our model using micro-level data on research expenditures by French firms. Our key finding is that uniform research subsidies can accentuate the dynamic misallocation in the economy by oversubsidizing applied research. Policies geared towards public basic research and its interaction with the private sector are significantly welfare-improving.

Keywords: Productivity, innovation, basic research, applied research, research and development, government spending, endogenous growth, spillover.

JEL classification: J82, L25, L50, O31, O38, O40

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1 Introduction

Fostering innovation is one of the primary objectives of economists and policymakers. The amount of resources invested in research is often at the heart of the debate over how to best achieve such a goal. Less well known, however, is what role the composition of research plays, particularly when considering the breakdown between basic and applied research. In this paper, we aim to fill this gap by studying the differential effects of basic versus applied research on aggregate innovation.

The distinction between basic and applied research is economically and empirically important. According to the National Science Foundation (NSF), basic research investment refers to a "systematic study to gain more comprehensive knowledge or understanding of the subject under study without specific applications in mind." Conversely, applied research is defined as a "systematic study to gain knowledge or understanding to meet a specific, recognized need." This distinction is empirically important since almost half of total research investment is allocated to basic research in countries such as France and the US.[1]

In a recent report by the US Congress's Joint Economic Committee, the issue of investment in basic research received fresh policy interest: the report argues that, despite its value to society as a whole, basic research is underfunded by private firms precisely because it is performed with no specific commercial applications in mind. It also states that the level of federal funding for basic research is "worrisome" and should be increased (Joint Economic Committee, 2016). Similarly, the NSF, the Sloan Foundation, and the National Bureau of Economic Research (NBER) have been proactively trying to draw attention to the declining government involvement in basic research funding.[2]

Despite clear empirical importance and considerable policy interest, the differential roles of basic and applied research in the growth process are still relatively unexplored from the macroeconomic perspective, and many related questions remain to be answered. How sizable are the spillovers from basic and applied research? What are the potential inefficiencies in a competitive economy? What are the appropriate government policies to mitigate these inefficiencies, and does the academic sector play a special role in determining the pace of innovation in the economy? This paper attempts to answer these important questions.

In order to understand the potential inefficiencies in research investment and to design appropriate industrial policies to address them, it is necessary to adopt a structural framework that explicitly models the incentives of private and public entities to engage in the different types of research. We propose a general equilibrium, multi-industry framework with private firms and a public research sector. In our model, basic research has two distinct features relative to applied research. First, as Nelson (1959) argues, "Successful basic research projects very often have practical value in many fields." In our setting, basic research can, therefore, generate spillovers that affect subsequent innovations both within and across industries. Applied research, on the other hand, generates innovations

[1] The OECD provides a breakdown of gross domestic expenditure on R&D expenditures by type of research: `https://stats.oecd.org/Index.aspx?DataSetCode=GERD_TORD#`.

[2] For instance, the NBER started a new initiative entitled the "The Science of Science Funding Initiative" (`http://admin.nber.org/drupal/SOSF`) funded by the Sloan Foundation.

within a targeted industry. Second, the potential returns from basic research depend on whether it was generated in the private or the public sector. In line with the "Ivory Tower" theory of academic research, basic research by private firms in our model will result in consumer products faster than that undertaken by public research labs.

These features of our model also reflect the important debate in science policy about the characteristics of basic and applied research as framed by Stokes (1997) in "Pasteur's quadrant: Basic Science and Technological Innovation." The academic sector represents pure scientific discovery without application in mind, best exemplified by Bohr's exploration of the structure of the atom. At the other extreme, lies the purely use-based –i.e., applied– private research, such as Edison's notable efforts in his Menlo Park laboratories. Finally, cutting across those extreme lines is "Pasteur's Quadrant," for work that is directly influenced in its course by the quest for fundamental understanding but inspired by an applied problem.[3] In our model, this role will be filled by private basic research. Prominent examples include the work by Nobel prize winners Arno Penzias and Robert Wilson at Bell Labs, doing, according to Rosenberg (1990), "basic research in astrophysics because of its relationship to the whole field of problems and possibilities in microwave transmissions, and especially the use of communication satellites for such purposes." Similarly, Nelson (1959) points out, "Carothers's famous work in linear superpolymers began as an unrestricted foray into the unknown with no particular practical objective in view. But the research was in a new field of chemistry, and Du Pont believed that any new chemical breakthrough would probably be of value to the company." In time, Carothers' research led to the discovery and use of nylon by Du Pont.

It is important to note that, given the novelty of our theoretical framework, the policy implications are not widely known. At its core, our theoretical framework builds upon the work of Klette and Kortum (2004), yet we extend it substantially in a variety of directions. Most importantly, we allow for the distinction between basic and applied research, as well as public and private research. These unique features allow us to be the first to simultaneously discuss the roles that spillovers generated by different research types and within different institutional arrangements, such as firms and universities, play for economic growth. It allows us to understand under what alternative specifications policy makers pushing for more public finding for private research could be right.

Our ultimate goal in this paper is to undertake a quantitative investigation of the impact of various innovation policies on the aggregate economy. To this end, we estimate the structural parameters of our model using French firm-level data for the period 2000-2006. Information about research investment comes from the annual R&D Survey conducted by the French Ministry of Research. The advantage of these data is that they include information on the basic and applied research expenditures of individual firms. To measure the breadth of a firm's activities, we combine two datasets ("Enquete Liaisons Financieres" (LIFI); "Enquete Annuelle des Entreprises" (EAE)) that allow us to precisely identify a firm's links to different industries not only through product lines within the same firm, but also through their ownership links. Finally, we use the balance sheet data from these firms

[3] The American physicist Gerald Holton called it "work that locates the center of research in an area of basic scientific ignorance that lies at the heart of a social problem."

to measure firm dynamics and NBER patent citation data to measure the quality of innovation. The final sample is composed of 13,708 firm-year observations.

These features of our data allow us to estimate the key spillovers involved in research. First, we measure the cross-industry spillovers associated with basic research through the investment choices of multi-industry firms. According to Nelson (1959), "a broad technological base insures that, whatever direction the path of research may take, the results are likely to be of value to the sponsoring firm."[4] In our data, we confirm that the proportion of basic research investment increases as the scope of a firm's activities expands. In our analysis, we provide extensive robustness checks of our estimates with respect to confounding factors arising from firm heterogeneity, industry heterogeneity, and measurement error. Second, we use citation information from the patent data to empirically measure the quality (step size) for basic and applied innovation, as well as their importance for follow-up innovations (Hall, Jaffe, and Trajtenberg, 2001). Note that, to ensure an exact correspondence between theory and empirics, we extend our theoretical framework to incorporate patent citations. In particular, in the model, patents are generated through successful research and subsequently cited by follow-up patents. Finally, to further inform the model parameters on firm dynamics, we simultaneously target a rich set of firm characteristics related to multi-industry presence, profitability, age, and entry/exit patterns. We discuss their identification through a set of comparative statics and untargeted moments.

Our main result is the quantification of the inefficiencies due to dynamic misallocation in research. We find that 68% of spillovers from basic research across industries are not internalized. As a result, there is a dynamic misallocation of research efforts, which reduces welfare by 4.6 percentage points in consumption-equivalent terms. One striking feature of the solution to the social planner's problem is that the fraction of resources devoted to research activities is not substantially greater than in the decentralized equilibrium. Indeed, the dominant misallocation here is not that between production and research, as is common in this class of models, but among the various types of research activities, in this case, applied and basic innovation. Another striking feature is that, in the case of applied innovation, there is actually an *overinvestment* in the decentralized economy due to the strategic complementarity between basic research spillovers and the returns to applied research.

This raises an important question: to what extent can public policies address this inefficiency? The first policy we analyze is a uniform research subsidy to private firms. In this environment, subsidizing overall private research is ineffective since this policy exacerbates the existing overinvestment in applied research arising from excessive competition. Therefore, the welfare improvements from such a subsidy are limited unless the policymaker is able to discriminate between types of research projects at the firm level. Thus, we consider a type-dependent research subsidy and find that the optimal policy is to subsidize basic research by 49% and applied research by 11%. While the type-dependent subsidy is a promising policy tool to increase welfare, one concern is that it might be difficult to implement. As Rosenberg (1990) notes, it is not always clear whether or not a specific undertak-

[4]Specific examples include Gibb's law of phases, which has been applied to activities ranging from petroleum refining to metal ore separation; or the development of the laser in 1964, which was used progressively in printers (1969), fiber optic communications (1970), bar code scanners (1974), vision correction surgery (1987), and DVD players (1995).

ing should be considered basic research. This might lead firms to misclassify research investment to exploit the differential subsidy rates. Therefore, we compute the optimal type-dependent policy under different degrees of misclassification. Importantly for policymakers, the welfare gains arising from a type-dependent subsidy are robust to a substantial amount of misclassification. Loosely speaking, the welfare gains associated with a type-dependent research subsidy are robust for levels of misclassification below 50%.

The appendix features an extensive set of theoretical and quantitative robustness checks. We re-estimate and evaluate the policy implications of the model with respect to industry heterogeneity and persistent firm types. We also re-estimate the model eliminating private basic research but maintaining public basic research; and eliminating both private and public basic research altogether. We then consider robustness checks with respect to the diffusion of public basic research and public research productivity. A final set of robustness checks relaxes assumptions about the spillover structure of applied research, and reevaluates the identification of the Constant Relative Risk Aversion (CRRA) parameter.

Related Literature Our main contribution is to the macro literature on endogenous technical change. Although policymakers have considered the different characteristics of basic and applied research, as well as public and private research, to be of first-order importance, these issues have received insufficient attention from the economic growth literature. In particular, models of endogenous technological change (see Aghion, Akcigit, and Howitt (2014) for a survey) mainly consider a uniform type of (applied) research and ignore basic research investment in the economy. A few exceptions are Aghion and Howitt (1996, 2009), Cozzi and Galli (2009, 2014), Gersbach, Schneider, and Schneller (2013), Gersbach and Schneider (2015), Morales (2004), and Mansfield (1995), all of whom consider theoretical models with both basic and applied research investment.

We contribute to this mostly-theoretical literature in various ways. First, we build a model with rich "firm dynamics" that is estimated with new firm-level micro data on firms' research investment.[5] To the best of our knowledge, ours is the first study to map a model with basic and applied research to firm-level data. In addition to including the private investment in basic research, we enrich the analysis of the distinct features of basic research by introducing a novel method to identify within- and cross-industry spillovers. Second, ours is the first framework that allows for a distinction between basic and applied research by private firms. Thanks to this feature, we are able to allow for differential efficiency properties of private basic versus applied research investments. One of our main results shows that innovation policies such as uniform research subsidies are ineffective since they oversubsidize applied research and undersubsidize basic research. Third, we complement this setting by also considering public research labs. This feature allows us to analyze the potential substitutability or complementarity between public and private research investments.

The paper also relates to the micro literature on innovation policy that discusses and quantifies the

[5]See Section 1.4 in Aghion, Akcigit, and Howitt (2014) for a detailed account of "realistic firm dynamics" in endogenous growth.

distinction between basic and applied research. Consistent with our framework, Nelson (1959) argues that a key distinction between these two types of research lies in the breadth of the potential discoveries. At the same time, Rosenberg (1990) provides a skeptical view on the possibility of categorizing research on the basis of the motives of the person performing it. We consider the associated measurement issue from within the context of our model and quantitative results. Cohen and Klepper (1992) focus on the relationship between firm size and R&D. In particular they argue that "social advantages of large firm size stem from the idea that large firms possess an advantage in appropriating the returns from innovation, due, in part, to imperfections in the market for information. This causes the returns to large firms to conduct more socially desirable R&D than smaller firms." In this paper, we identify the economic scope of a firm as a factor attenuating the appropriability problem of research, and we conceptually disentangle it from a pure size effect. Empirically, several important papers (Mansfield, 1980; Mansfield, 1981; Link, 1981; Griliches, 1986) show that the distinction between basic and applied research is quantitatively important. These papers typically evaluate the importance of basic and applied research by relating it to production data and estimating its output elasticity or rate of return from an extended Cobb-Douglas production function. The common conclusion from those papers is that basic research seems to command a significant "premium"– i.e., that its contribution to firm productivity is significantly larger than that of applied research.

Methodologically, our paper is related to the growing branch of endogenous growth with firm dynamics that estimates these models structurally using micro data. For instance, Lentz and Mortensen (2008, 2016) use a panel of Danish firms; Acemoglu, Akcigit, Alp, Bloom, and Kerr (2018), Akcigit and Kerr (2018), and Garcia-Macia, Hsieh, and Klenow (2019) use US Census of Manufacturing; Peters (2015) uses Indonesian micro data; and Ates and Saffie (2014) use Chilean firm-level data to estimate enriched versions of the quality-ladder models. A number of papers also study the role of innovation policy in a similar class of models. For instance, Atkeson and Burstein (2019) study the impact of policy-induced changes in innovative investment by firms on growth in aggregate productivity. Akcigit, Hanley, and Stantcheva (2016) use a mechanism design approach to solve for the optimal design of innovation policy using a quality-ladder model with asymmetric information on firm types. Finally, Garicano, Lelarge, and Van Reenen (2016) use firm-level data to study the impact of size-dependent policies on misallocation of factors of production in France. These papers do not consider the distinction between basic and applied research, whereas our focus is on estimating the associated spillovers using French firm-level data and design-relevant policies around these spillovers.

Consistent with our results, some papers show that the speed of technology diffusion is linked to the ability of inventors to utilize ideas for production (Akcigit, Celik, and Greenwood (2016)), or to the existing patent rights over those technologies (Galasso and Schankerman (2015)). In the same spirit, Bloom, Schankerman, and Van Reenen (2013) identify technology and product market spillovers for US manufacturing firms. They show that small firms generate lower social returns to R&D because they operate in technological niches. Our paper suggests that their result can be rationalized by these firms' lower incentives to invest in basic research that is more difficult to appropriate. In our framework, more public basic research investment stimulates more private applied research. In that regard, our results parallel the findings of Cozzi and Impullitti (2010) and Impullitti (2010), who show that increases in the technological content of public spending encourages more private R&D spending in the US.

The rest of the paper is organized as follows. In Section 2, we introduce the model, characterize its dynamic equilibrium, and discuss the main mechanisms. In Section 3, we present and discuss the empirical patterns that provide the basis for the structural estimation. Section 4 describes the estimation and identification of the model. Section 5 provides a detailed discussion of the welfare effects of various policies on the decentralized economy. Section 6 concludes.

2 Theory: Growth with Basic and Applied Research

Our theoretical framework will depart from standard endogenous growth models in a number of ways. First, we introduce a distinction in the appropriability of innovations from basic and applied research. Following the influential literature on basic science, we consider the possibility that basic research generates not only spillovers within an industry, but also across industries (Nelson (1959)). Thus, we model an economy in which firms can operate in multiple industries, a feature that endogenously generates incentives for firms to invest in basic research.

A second key feature of our model is the distinction between embodied and disembodied knowledge in the economy. Both private firms and public research labs are investing in basic research in this economy. However, successful basic research in the private sector is more likely to be immediately turned into a consumer product (embodied), as opposed to simply increasing the stock of knowledge available for future innovators (disembodied). This will induce a delay in the effect of public basic research. This *ivory tower* nature of academic research has been widely discussed in academic and policy circles, with a formal analysis being provided by Aghion, Dewatripont, and Stein (2008).[6]

2.1 Basic Environment

2.1.1 Production

Production is divided into three major levels: *upstream*, *midstream*, and *downstream*. First, the upstream level produces intermediate goods (y_{ij}); next, these goods are used to produce industry aggregates (Y_i) in the midstream level; and, finally, the downstream level combines these industry aggregates into the final good (Z). We will now describe them in detail.

Downstream Production The final good $Z(t)$ is produced in the downstream by infinitely many competitive firms that combine inputs from M different industries according to the following Cobb-Douglas production function:

$$Z(t) = \prod_{i=1}^{M} Y_i(t)^{\frac{1}{M}}. \tag{1}$$

[6]In their model, the academic sector is a precommitment mechanism that allows scientists to freely pursue their own interests. Consistent with our model, academic scientists may, therefore, end up working on projects with little immediate economic value. An important difference, however, is that in their setup, there is full appropriability of the innovation. We relax this assumption and show that it generates a delay in the innovation process with consequences for economic growth.

In this production function, $Y_i(t)$ is the aggregate output from industry $i \in \{1, ..., M\}$. The economy consists of $M \in \mathbb{N}^+$ industries. In the context of firm-level data, each industry i can be thought of as a different 1-digit Standard Industrial Classification (SIC) code and $Z(t)$ is simply the aggregate GDP of the economy.[7] We normalize the price of the final good to 1 at every instant t without any loss of generality. For notational simplicity, time subscripts will henceforth be suppressed.

Midstream Production Each industry aggregate Y_i is produced competitively, combining inputs from a continuum of product lines. Let y_{ij} denote the production of upstream good j in industry i by the firm that has the best technology in that product line. Industry aggregate i is produced according to the following CES production function:

$$Y_i = \left[\int_0^1 y_{ij}^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{2}$$

Upstream Production In product line j, the firm that has the latest (and also the best) technology produces as a monopolist according to the following linear production technology that takes only labor as an input:

$$y_{ij} = q_{ij} l_{ij}, \tag{3}$$

where $q_{ij} > 0$ is the labor productivity associated with product line j, and l_{ij} is the number of production workers employed. Let us denote the wage rate in the economy by w in terms of the final good. The specification in (3) implies that each product y_{ij} has a constant marginal cost of production $w/q_{ij} > 0$. We denote the productivity index of industry i by

$$\bar{q}_i \equiv \left(\int_0^1 q_{ij}^{\varepsilon-1} dj\right)^{\frac{1}{\varepsilon-1}}. \tag{4}$$

Definition of a Firm In this model, as in Klette and Kortum (2004), a firm is defined as a collection of product lines in which it is the lead producer. These product lines can come from multiple industries. In what follows, $m_f \in \{1, .., M\}$ will denote the number of industries in which the firm actively operates; $n_{if} \in \mathbb{N}^0$ will denote the number of product lines firm f owns in a given industry i (e.g., $n_{if} = 0$ means no presence of firm f in industry i); and n_f will stand for the total number of product lines owned by the firm and will satisfy $n_f \equiv \sum_{i=1}^{M} n_{if}$. Henceforth, for notational tractability, we will drop the firm index f, when it creates no confusion.

A firm's payoff in a given product line j in industry i depends on its productivity level q_{ij}. Therefore, the payoff-relevant state of a firm is denoted by

$$\mathbf{q} = (\mathbf{q}_1, \mathbf{q}_2, \ldots, \mathbf{q}_m),$$

[7]Note that we introduce this multi-industry structure in order to model cross-industry spillovers. To avoid any additional theoretical complications, we will focus on symmetric equilibria in which industry aggregates assume a common value.

where $\mathbf{q}_i = \left\{q_{i,j_1}, q_{i,j_2}, \ldots, q_{i,j_{n_i}}\right\}$ is a multi-set keeping track of all the productivity levels of the firm in industry i, where it has the best technology.[8]

Example 1. *An example is helpful to summarize the description thus far. Figure 1 illustrates an economy that consists of $M = 4$ industries and a firm that operates in $m = 3$ industries ($i = 1$, $i = 3$, and $i = 4$). It has $n_1 = 3$ product lines in industry $i = 1$, $n_3 = 2$ lines in $i = 3$, and $n_4 = 1$ line in $i = 4$; thus, firm f has $n = 6$ product lines in total. This firm does not currently operate in industry $i = 2$.*

Figure 1: Payoff-Relevant State $\mathbf{q}$

$$\mathbf{q}(t) \equiv \left\{ \begin{array}{cccc} \underline{i=1} & \underline{i=2} & \underline{i=3} & \underline{i=4} \\ q_{1,j_1}(t) & & q_{3,j_1}(t) & q_{4,j_1}(t) \\ q_{1,j_2}(t) & & q_{3,j_2}(t) & \\ q_{1,j_3}(t) & & & \end{array} \right\}.$$

A firm's portfolio of products will expand through successful innovation. Likewise, it will lose product lines when other firms or potential entrants successfully innovate on one of its product lines (thus surpassing it). These innovations will be the source of economic growth in this economy. The next subsection will describe the details of the innovation technology.

2.1.2 Innovation and Technological Progress

In this economy, there are two types of innovations (basic and applied) and two different groups of agents (private and public sectors) generating productivity growth. As Nelson (1959) and Aghion and Howitt (1996) describe it, fundamental advances in technological knowledge come through basic innovation and open up windows of opportunity for future research. Applied innovation builds on these existing basic innovations, thus realizing these opportunities.

Research by Private Firms For the innovation production function, we will follow the literature,[9] but extend it to both (a)pplied and (b)asic research. More specifically, firms undertake s-type research ($s \in \{a, b\}$) operating with the following convex cost function:

$$h_s(s_i) = \xi_s s_i^{\nu_s} \tag{5}$$

in terms of researchers per product line, where $s_i \in \{a_i, b_i\}$ is the s-type innovation Poisson flow per product line, ξ_s is a scale parameter, and $\nu_s > 1$ is the cost elasticity.[10] Note that this production

[8]A multi-set is a generalization of a set that can contain more than one instance of the same member. For example, given $j \neq j'$, a multiset $\mathbf{q}_{if}$ can contain $q_{if}(j)$ and $q_{if}(j')$ regardless of whether $q_{if}(j) = q_{if}(j')$.

[9]See, for instance, Klette and Kortum (2004), Lentz and Mortensen (2008), Acemoglu, Akcigit, Hanley, and Kerr (2016), and Acemoglu, Akcigit, Alp, Bloom, and Kerr (2018).

[10]This per-product cost function can be derived from a firm-level production function that combines (in a Cobb-Douglas manner) the number of product lines n_i and total research employment H_i to yield total innovation output S_i, as in

$$S_i = \xi_s^{-\frac{1}{\nu_s}} n_i^{1-\frac{1}{\nu_s}} H_{s,i}^{\frac{1}{\nu_s}}.$$

is specified at the product line level, and a firm's total innovative capacity scales additively across product lines.

Both applied and basic research are *directed* toward particular industries but *undirected* within those industries. In other words, once a firm chooses s_i, the realization of innovations will take place on a random product within industry i. Once this occurs, the newly innovating firm will replace the incumbent firm as the monopolist.[11]

In our data, some firms do not invest in basic research. To capture this fact, we generalize the basic research technology by introducing a fixed cost of doing basic research. At each instant, a firm with n product lines draws a fixed labor cost of doing basic research $n\phi \geq 0$, where ϕ is distributed according to the distribution $\mathcal{B}(\cdot)$ having mean $\bar{\phi}$.

2.1.3 Spillovers of Basic Research

Let $q_{ij}(t)$ be the highest productivity technology for producing j in industry i. When a firm that operates in industry i produces a basic innovation in industry i and product line j, the same firm uses this basic knowledge for production and patents this new high-value technology. As a result, the firm improves $q_{ij}(t)$ by $\eta\bar{q}_i(t)$

$$q_{ij}\left(t+\Delta t\right) = q_{ij}\left(t\right) + \eta\bar{q}_i(t), \tag{6}$$

where $\eta > 0$ is the step size drawn from exponential distributions with mean $\bar{\eta}$, and $\bar{q}_i$ is the productivity index defined in equation (4). Notice that this additive formulation features mean reversion in terms of proportional step sizes and, therefore, generates an invariant firm-size distribution over the normalized quality $(q_{ij}/\bar{q}_i)$. When the firm produces this new innovation, it adds this product line with the productivity improvement to its portfolio $\mathbf{q}(t+\Delta t) = \mathbf{q}(t) \cup \left\{q_{ij}(t+\Delta t)\right\}$, which generates per-period profit of $\pi\left(q_{ij}(t+\Delta t)\right)$. For instance, when the firm in Figure 1 is successful with its basic research investment in industry $i=1$, its number of products goes up from $n_1 = 3$ to $n_1 = 4$, as illustrated in Figure 2:

FIGURE 2: IMPACT OF BASIC INNOVATION IN INDUSTRY $i=1$

$$\begin{array}{cccc} \underline{i=1} & \underline{i=2} & \underline{i=3} & \underline{i=4} \end{array}$$

$$\mathbf{q}(t) \equiv \left\{ \begin{array}{cccc} q_{1,j_1}(t) & & q_{3,j_1}(t) & q_{4,j_1}(t) \\ q_{1,j_2}(t) & & q_{3,j_2}(t) & \\ q_{1,j_3}(t) & & & \\ q_{1,j_4}(t)+\eta\bar{q}_1(t) & & & \end{array} \right\}.$$

Basic research features two potential spillovers: (a) *cross-industry spillover* and (b) *within-industry spillover*. These spillovers lie at the heart of our analysis; therefore, we will now discuss each in more detail.

[11] Note that the implicit assumption here is that patents grant a monopoly to the leading firm but provide no blocking power against new innovators.

Cross-Industry Spillover from Basic Research The characteristic feature of basic research that we wish to capture is that it often has applications in many industries other than the one for which it was originally intended. Therefore, we will assume that when a basic innovation occurs, it applies with probability one to the target industry i (as illustrated above in Figure 2) and generates a random number of "additional" spillovers in other (or the same) industries. We denote the expected number of such additional spillovers with $p > 0$. Thus, p is our measure of the intensity of cross-industry spillovers. Note that not every spillover can be utilized by the firm. When a firm generates some basic knowledge, it can turn this into an immediate application only in the industry in which it has working knowledge. This structure captures the hypothesis put forth by Nelson (1959).

Going back to our earlier example in Figures 1 and 2, we can now illustrate how cross-industry spillovers operate. Thanks to its success, the firm has already produced a new basic innovation in industry $i = 1$. Assume, now, that this basic innovation has "additional" spillovers in industries 1, 2, and 3 but not in $i = 4$. Since firm f already operates in industries 1, 3, and 4, it can turn this new finding into patentable products in industries 1 and 3, as illustrated in Figure 3.

Figure 3: Cross-industry Spillover into Industry $i = 3$

$$\mathbf{q}(t) \equiv \left\{ \begin{array}{cccc} \underline{i=1} & \underline{i=2} & \underline{i=3} & \underline{i=4} \\ q_{1,j_1}(t) & & q_{3,j_1}(t) & q_{4,j_1}(t) \\ q_{1,j_2}(t) & & q_{3,j_2}(t) & \\ q_{1,j_3}(t) & & q_{3,j_3}(t) + \eta \bar{q}_3(t) & \\ q_{1,j_4}(t) + \eta \bar{q}_1(t) & & & \\ q_{1,j_5}(t) + \eta \bar{q}_1(t) & & & \end{array} \right\}.$$

We model the randomly realized number of spillovers generated with a geometric distribution. That is, the probability of receiving $n \geq 0$ spillovers (in addition to the original innovation) is given by

$$F^n = (1-s)s^n \quad \text{where} \quad s = \frac{p}{1+p}. \tag{7}$$

Note that the number of spillovers has expectation p by construction. Recall that m denotes the number of industries in which a firm has working knowledge. As we show in Appendix D, this leads to the following distribution of utilized spillovers (k) for a firm with working knowledge in m industries:

$$F_m^k = (1-s_m)s_m^k \quad \text{where} \quad s_m = \frac{sm}{sm + (1-s)M} \leq s. \tag{8}$$

Consequently, the expected number of utilized spillovers for the firm is

$$\mathit{Utilized\_Spillovers}(m) = p \times \frac{m}{M}.$$

This highlights the well-known *appropriability problem* of basic research. As Nelson (1959) puts it, firms that have "*fingers in many pies*" – i.e., m in our model – have a higher probability of using the results of basic research. Hence, a broad technological base increases the probability of benefiting from successful basic research.

Within-Industry Spillover from Basic Research *Applied research* makes use of the *within-industry* spillover from basic research and builds on the existing latest basic technological knowledge in a product line. The productivity of each applied innovation is a function of the depreciation of the latest basic technology. If the latest basic knowledge in j is undepreciated (i.e., still hot), a successful applied innovation will benefit from it and improve the latest productivity $q_{ij}(t)$ of that product line by $\eta\bar{q}_i(t)$, as in expression (6). If the latest basic technology of the product line is depreciated (i.e., cold), a successful applied innovation will improve the latest productivity only by an amount proportional to λ, so that

$$q_{ij}(t+\Delta t) = \begin{cases} q_{ij}(t) + \eta\bar{q}_i(t) & \text{if } j \text{ is hot} \\ q_{ij}(t) + \lambda\bar{q}_i(t) & \text{if } j \text{ is cold} \end{cases} \tag{9}$$

Note that the step sizes η and λ are drawn each time from an exponential distributions with mean $\bar{\eta}$ and $\bar{\lambda}$.

We assume that a new basic technology depreciates (innovations run into diminishing returns) at a Poisson rate $\zeta > 0$. On the other hand, a new basic innovation reactivates the product line until the next time that it cools down again. If the product line was already hot, there is no additional effect. Let us denote the arrival rate of basic innovations to product lines by τ_b. Then, during a small time interval Δt, each product line j will be subject to the transition rates denoted in Table 1:

TABLE 1: TRANSITION MATRIX FOR WITHIN-INDUSTRY SPILLOVERS

	hot	**cold**
hot	$1-\zeta\Delta t$	$\zeta\Delta t$
cold	$\tau_b\Delta t$	$1-\tau_b\Delta t$

Public Basic Research In our model, the academic sector will be the other source of basic knowledge creation. One of the main tasks of public research labs in an economy is to produce the necessary basic scientific knowledge that will be part of the engine for subsequent applied innovations and growth. We assume that the public research sector consists of a measure U of research labs *per industry*. Each lab receives the same transfer $\bar{R}$ from the government to finance its research, which results in an overall funding level of $R = \bar{R} \times U \times M$.

We assume that each public research lab generates a flow rate of u by hiring $h_b(u)$ researchers with the same basic research technology as a one-product firm in (5), so that $h_b(u) = \xi_b u^{\nu_b}$.[12] Additionally, as with private basic research, they face a per-lab fixed cost $\bar{\phi}$ of operation. This specification implies that the government can affect the basic knowledge pool in the economy through the amount of funds R allocated to the academic sector. The flow rate of basic innovation from the academic sector will

[12] In reality, public research labs may have a different research technology than private labs. However, obtaining data on both the inputs and outputs of individual public labs is difficult. The separate estimation of public and private innovation production functions is left for future research.

satisfy

$$\bar{R} = w[h_b(u) + \bar{\phi}], \tag{10}$$

where u is the academic basic innovation flow *per lab*. In this economy, R is a policy lever controlled by the policymaker. As with private firms, each basic innovation has the same within- and cross-industry spillover structure. Note that the equilibrium fraction of hot product lines α will be determined by the aggregate rates of public (u) and private (b_m) basic research, as well as by the cooldown rate (ζ). The key difference between private and public basic research is the "Ivory Tower" nature of the latter, meaning that these innovations by public labs will turn into output only upon a subsequent private applied innovation.[13]

Research and Innovation Policy In the baseline model, we consider proportional subsidies to research spending. We allow for there to be differential subsidy rates ψ_a and ψ_b for applied and basic, respectively. In the policy analysis, we consider optimal policies for the case where there is a uniform subsidy ($\psi_a = \psi_b$) and where the subsidy rates can differ ($\psi_a \neq \psi_b$).

2.1.4 Entry, Exit, and Industry Dynamics

Entry There is a mass E of outside entrants per industry. The research technology for a single outside entrant is assumed to be the same as that of applied innovation for a firm with a single product line. Thus, if an outside entrant wishes to produce a flow probability a_e of entry, they must hire $h_a(a_e) = \xi_a a_e^{\nu_a}$ researchers.

Exit In our model, there will be both endogenous and exogenous channels for firm exit. First, a firm that loses all of its product lines to other competitors will have a value of zero and, thus, will exit. Second, each firm has an exogenous death rate $\kappa > 0$. When this occurs, the firm sells all of its product lines to random firms at a “fire sale” price $\mathcal{P}$.[14] On the flip side, firms will receive a buyout option with a probability that is proportional to their number of products.

Expansion into New Industries The economy features $E \times a_e$ flow of entry at any instant. We will assume that a ς fraction of new entrants will meet a randomly selected incumbent firm. Thus, an incumbent will have a flow rate of incoming buyout offers

$$x \equiv \varsigma E a_e / F,$$

where F is the equilibrium measure of firms. If $\bar{n}$ denotes the average number of product lines per firm, then $F = 1/\bar{n}$. Clearly, this new company will be from a new industry with probability

[13]It is important to note that we assume that innovation done by public labs is turned into consumer products only upon subsequent innovation by private firms. The lag between the creation of publicly funded innovations and actual goods production is empirically shown in a large literature (e.g., Rosenberg and Nelson (1994) and Mowery, Nelson, Sampat, and Ziedonis (2004), among several others). This important issue is generally overlooked in the theoretical growth literature. Inclusion of this feature generates some new and interesting dynamics, such as the importance of involvement of the private sector in basic research.

[14]The exact value of this price will not play any role in the equilibrium determination.

$(1-m/M)$ or from an industry that already exists in the incumbent's portfolio with probability m/M. Our goal is to keep the M&A margin as tractable as possible, and we will achieve this by assuming that the M&A price that the incumbent firm has to pay is equal to the full surplus of the new merger.[15]

Labor Market Labor is split between production (L_p) and research labor. Research labor can be further subdivided into that devoted to private basic research (L_b), public basic research (L_u), private applied research (L_a), and firm entry (L_e). There is a unit mass of workers per industry, meaning the total labor supply consists of M workers. The labor market clearing condition is given by

$$M = L_p + L_b + L_a + L_e + L_u.$$

Household Problem Finally, we close the model by describing the household problem that determines the equilibrium interest rate in this model. The household consumes the final good and maximizes the following lifetime utility

$$W_0 = \int_0^\infty \exp\left(-\delta t\right) \frac{C(t)^{1-\gamma} - 1}{1-\gamma} dt, \tag{11}$$

where $C\left(t\right)$ is consumption at time t; γ is the constant relative risk-aversion parameter; and δ is the discount rate. The household owns all the firms in the economy, which generates a risk-free flow return of r in aggregate. The household also supplies labor in the economy, through which it earns wage rate $w\left(t\right)$. Finally, the household pays a lump-sum tax $T\left(t\right) \geq 0$ every instant. Thus, the household's intertemporal maximization is simply to maximize (11) subject to the following budget constraint:

$$C\left(t\right) + \dot{A}\left(t\right) \leq r\left(t\right) A\left(t\right) + Mw\left(t\right) - T\left(t\right),$$

where $A\left(t\right)$ is the asset holdings of the household.

2.2 Equilibrium

In this section, we characterize the dynamic equilibrium of our model. Our focus is on a symmetric balanced-growth-path (SBGP) equilibrium in which all industries start with the same initial conditions at time $t = 0$, and all aggregate variables grow at the same endogenous rate g.

In this model, three variables affect the payoff of the firm: the number of product lines n; the number of industries m, and the relative productivity

$$\hat{q}_{ij} \equiv q_{ij}/\bar{q}_i \tag{12}$$

of its product lines, which is the absolute productivity in line j normalized by the productivity index $\bar{q}_i$ in industry i. Note that this implies that successful innovation leads to a constant additive increment

[15]The resulting invariant joint distribution $\Gamma_{m,n}$ over multi-industry presence m and firm product count n is described in Appendix D

in $\hat{q}$ space, that is, $\hat{q} \rightarrow \hat{q} + s$ where $s \in \{\eta, \lambda\}$ depends on the type of innovation. Thus, each incumbent firm is characterized by its state $k \equiv (\hat{\mathbf{q}}, n, m)$.

Of particular interest is the distribution of firms $\Gamma_{n,m}$, which denotes the mass of firms operating in m industries and having n product lines in total. This will sum not to one but to the total number of firms, which is endogenous. We also denote the mass of product lines owned by firms in m industries by μ_m, which can be computed from the joint distribution using $\mu_m \equiv \sum_{n=1}^{\infty} n \cdot \Gamma_{m,n}$. This will in fact sum to one, as the total mass of product lines in each industry is also one.

Given a government policy sequence $[T(t)]_{t=0}^{\infty}$, an SBGP equilibrium is composed of a sequence of intermediate-good quantities; prices; the basic and applied innovation rates of private firms and entrants; the wage rate and interest rate; the joint distribution of multi-industry presence and product count; hot and cold product line productivity distributions $(\mathcal{F}_H(t), \mathcal{F}_C(t))$; and the fraction of hot product lines – i.e., $[y_k(t), p_k(t), b_k(t), a_k(t), a_e(t), w(t), r(t), \Gamma_{m,n}(t), \mathcal{F}_H(t), \mathcal{F}_C(t), \alpha(t)]_{t=0}^{\infty}$ – such that all firms choose quantity and price to maximize their profits; incumbent and entrant firms invest in research to maximize their firm value; the labor market clears; the household maximizes its discounted sum of future utilities; and the distributions satisfy the relevant flow equations.

Solution of the Model In this setting, the intermediate producer with the state-of-the-art technology $\hat{q}$ operates as a monopolist. Thus, each new innovation has an element of creative destruction, wherein the new innovator replaces the old incumbent using superior technology. In order to avoid the possibility of limit-pricing, we follow Acemoglu, Akcigit, Alp, Bloom, and Kerr (2018) and Akcigit and Kerr (2018), and assume that the current incumbent and any former incumbents in the same line (with lower quality than the current incumbent) enter a two-stage price-bidding game. In the first stage, each firm pays a fee of $\epsilon > 0$, which is arbitrarily close to 0. In the second stage, all firms that paid the fee announce their prices. Due to Bertrand competition in the second stage, only the most productive firm will be able to make any sales and profits, and thus, only this firm will pay the cost ϵ, enter the pricing game, and operate with monopoly pricing.

Standard profit maximization delivers familiar equilibrium price and quantities (interested readers are referred to Appendix Section D for the detailed derivations and results). A monopolist's quantity is increasing and price is decreasing in the relative productivity $\hat{q}$ of the product line. The equilibrium profits of the monopolist are increasing in its relative productivity $\hat{q}$ and the average market size Z/M:

$$\pi(\hat{q}) = \frac{\hat{q}^{\varepsilon-1}}{\varepsilon}\frac{Z}{M}. \tag{13}$$

From this expression, for the flow profits at the product line level, we can then construct a value function describing the overall value of a firm with a portfolio $\hat{\mathbf{q}}$ of product lines operating in m industries. This rather ungainly object is presented in full in Appendix D. However, the intuition for its construction is fairly straightforward. There is a flow value associated with profits net of various types of R&D spending. On top of this, there are a variety of events that can happen to the firm, such as gaining a new product line on successful research, expanding into a new industry, or losing a product line to a competitor who has innovated. Proposition 1 summarizes our main result concerning firm value.

Proposition 1. *Let the value of a firm with a productivity portfolio* $\hat{\mathbf{q}}$ *in* m *industries be denoted by* $\mathcal{V}(\hat{\mathbf{q}}, m)$*. This value is equal to*

$$\mathcal{V}(\hat{\mathbf{q}}, m) = \frac{Z}{M}\left[\sum_{\hat{q}\in\hat{\mathbf{q}}} V(\hat{q}) + nV_m\right],$$

where

$$V(\hat{q}) = \frac{\hat{q}^{\varepsilon-1}}{\varepsilon\left[r+\tau+\kappa+g\left(\varepsilon-2\right)\right]},$$

and

$$(r-g)\,V_m = \max_{a,b}\left\{\begin{array}{c} -\tilde{w}\left[(1-\psi_a)h_a(a) + (1-\psi_b)[h_b(b) + \mathbf{1}_{(b>0)}\phi]\right] \\ +a\left[\alpha V^H + (1-\alpha)\,V^C + V_m\right] + b\left(1+p\frac{m}{M}\right)\left[V^H + V_m\right] \\ +x\left(1-\frac{m}{M}\right)\left[V_{m+1} - V_m\right] - \tau V_m + \kappa\mathbb{E}_{\hat{q}}V(\hat{q}_t) \end{array}\right\}. \tag{14}$$

The analogous production values are defined as $V^H \equiv \mathbb{E}^H_{\hat{q},\eta}V(\hat{q}+\eta)$ *and* $V^C \equiv \mathbb{E}^C_{\hat{q},\lambda}V(\hat{q}+\lambda)$*.*

Proof. See Appendix D ■

This important result has a number of implications. First, the value of a firm has a tractable additive form across product lines. Moreover, the firm value has two major components: the first component is the production value $V(\hat{q})$, which simply computes the sum of the future discounted profits where the effective discount rate takes into account the rate of creative destruction τ, the exogenous destruction rate κ, and the obsolescence of the relative productivity $\hat{q}$ due to the growth of $\bar{q}$. The second component is the R&D option value V_m, which is a direct function of the multi-industry presence due to the associated internalization of spillovers. Finally, because of the stochastic nature of step sizes, the expectations now integrate over the productivity (which is type specific) and step size.

Because the value function is additive across product lines, so then are innovation rates. Now we can express the first-order condition for basic innovation at the product line level with

$$b_m = \left[\frac{\left(1+p\frac{m}{M}\right)\left(V^H + V_m\right)}{(1-\psi_b)\nu_b\zeta_b\tilde{w}}\right]^{\frac{1}{\nu_b-1}}$$

The most important result here is the fact that the cross-industry spillover is stronger when a firm is operating more industries; thus, we would expect that basic research investment is increasing in the multi-industry presence of the firm. The strength of this positive relationship will be governed mainly by the probability of the cross-industry spillover parameter p. Because of the fixed cost to undertaking basic research ϕ, some labs will decide not to operate if their draw is too high. Let this threshold value be denoted ϕ_m. Furthermore denote the resulting fraction of labs that operate be denoted $P_m = \mathbb{P}[\phi < \phi_m]$.

Both private firms and public research labs are generating basic research in this economy. It is useful to break down total basic research into its embodied and disembodied components. The distinction is based on whether the basic knowledge is immediately turned into a consumer product (embodied) or simply added to the stock of knowledge available for future innovators (disembodied). We obtain the following aggregates:

$$\begin{aligned}
\text{Embodied:} \quad \tau_b^e &\equiv \sum_{m=1}^{M} \mu_m P_m (1+\rho_m) b_m \\
\text{Disembodied:} \quad \tau_b^d &\equiv \sum_{m=1}^{M} \mu_m P_m (p-\rho_m) b_m + (1+p)u \\
\text{Total:} \quad \tau_b &\equiv \tau_b^e + \tau_b^d,
\end{aligned} \tag{15}$$

Recall that μ_m is the mass of product lines owned by firms operating in m industries μ_m. Then, τ_b^e and τ_b^d correspond, respectively, to the embodied and disembodied components of basic research. Note that the disembodied component includes both private spillovers that are unused and the results of public basic innovation. Finally, τ_b is simply the overall flow of basic innovation, including all spillovers.

Using this aggregate rate and the cooldown rate ζ, we can express the steady-state flow equation: the number of product lines that become hot must be equal to the number of product lines that cool down. In other words, we must have $\alpha\zeta = (1-\alpha)\tau_b$. As a result, the steady-state fraction of hot product lines is

$$\alpha = \frac{\tau_b}{\zeta + \tau_b}. \tag{16}$$

The share of hot product lines – those having basic knowledge that can be turned into better consumer products (α) – is increasing in the amount of basic research flow. This expression highlights the role of public policy in affecting the knowledge stock. The more money that is allocated to public basic research, the higher the basic research flow will be from public research labs (u), which will then increase the fraction of hot product lines through τ_b, as in (15) and (16).

Firms invest in applied research according to

$$a = \left[\frac{\alpha V^H + (1-\alpha) V^C}{(1-\psi_a)\nu_a \xi_a \tilde{w}} \right]^{\frac{1}{\nu_a - 1}}.$$

The crucial observation here is the complementarity between basic and applied research. In equilibrium, $V^H > V^C$, since hot product lines are associated with a larger step size η. Hence, if there are more hot product lines (a higher α), each firm increases its investment in applied research. This renders basic and applied research investments complementary. In particular, holding the wage rate fixed, higher public basic research investment encourages firms to invest more in applied research through higher within-industry spillovers.

Let us denote the aggregate rate of applied innovation by τ_a such that

$$\tau_a = \sum_{m=1}^{M} \mu_m a_m + E a_e. \tag{17}$$

Recall that τ_b^e denotes the arrival rate of embodied basic research, as defined in (15). Now we can denote the aggregate rate of *creative destruction* (the rate at which firms lose product lines to other firms) by τ:

$$\tau \equiv \tau_a + \tau_b^e. \tag{18}$$

Creative destruction is determined by the rate at which incumbents produce basic innovations that can be embodied into production immediately (τ_b^e), and by the rate at which incumbents and entrants produce applied innovations (τ_a). Now we are ready to state the following lemma:

Lemma 1. *Let $\mathcal{F}_H(\cdot)$ and $\mathcal{F}_C(\cdot)$ be the aggregate product cumulative measures by type (hot or cold). The flow equations for these objects are, respectively,*

$$\dot{\mathcal{F}}_H(\hat{q}) = -\tau\left[\mathcal{F}_H(\hat{q}) - \mathcal{F}_H(\hat{q}-\eta)\right] + \tau_b^e \mathcal{F}_C(\hat{q}-\eta) - \zeta\mathcal{F}_H(\hat{q}) + \tau_b^d \mathcal{F}_C(\hat{q}) + g\hat{q}[\partial\mathcal{F}_H(\hat{q})/\partial\hat{q}]$$
$$\dot{\mathcal{F}}_C(\hat{q}) = -\tau_a\left[\mathcal{F}_C(\hat{q}) - \mathcal{F}_C(\hat{q}-\lambda)\right] - \tau_b\mathcal{F}_C(\hat{q}) + \zeta\mathcal{F}_H(\hat{q}) + g\hat{q}[\partial\mathcal{F}_C(\hat{q})/\partial\hat{q}].$$

Proof. See Appendix D. ■

The labor market clearing condition can now be expressed in terms of the above endogenous variables. One additional relationship we will exploit is that between the mass of labor devoted to production and the normalized wage rate. This can be derived from the goods production specification (see Section D in the Appendix for its detailed derivation)

$$L_p = \frac{Z}{w}\left(\frac{\varepsilon-1}{\varepsilon}\right)$$

Using this and the symmetric nature of the equilibrium, we express the labor market clearing condition as an average over industries:

$$1 = \frac{1}{\tilde{w}}\left(\frac{1-\varepsilon}{\varepsilon}\right) + \sum_m \mu_m P_m\left[h_b(b_m) + \bar{\phi}_m\right] + U\left[h_b(u) + \bar{\phi}\right] + h_a(a) + Eh_a(a_e) \tag{19}$$

where $\bar{\phi}_m = \mathbb{E}[\phi|\phi < \phi_m]$. This expression equates the labor supply per industry (= 1 since the total labor supply is M) to labor demand for production workers; private basic researchers, which is a function of the multi-industry presence of the firms; public basic researchers, which is determined by public policy; incumbent applied researchers; and entrant basic researchers.

Finally, as derived in the Appendix, by imposing aggregate consistency with the goods production functions (2) and (1), we find that the aggregate output is

$$Z = \bar{q}L_p/M. \tag{20}$$

This expression simply says that the aggregate output is equal to the product of the number of workers employed for production and the aggregate productivity index of the economy. In an SBGP equilibrium, the labor allocated for production is constant. Therefore, the growth rate of aggregate output (and also output per worker) will be equal to the growth rate of the productivity index $\bar{q}$. The following proposition provides the exact growth rate of the productivity index.

Proposition 2. *In an SBGP, the growth rate of the productivity index is*

$$g = \frac{\tau_a\left[\alpha\mathbb{E}_{\hat{q}}^H\left(\hat{q}+\eta\right)^{\varepsilon-1} + (1-\alpha)\mathbb{E}_{\hat{q}}^C\left(\hat{q}+\lambda\right)^{\varepsilon-1} - 1\right] + \tau_b^e\left[\mathbb{E}_{\hat{q}}\left(\hat{q}+\eta\right)^{\varepsilon-1} - 1\right]}{\varepsilon-1}. \tag{21}$$

Proof. See Appendix D ■

This growth expression shows that the engines of economic progress include both applied and basic innovation. More important, the basic knowledge stock in the economy, represented by α, makes each applied innovation more valuable and contributes more to growth (since $\eta > \lambda$). This expression shows how public funding can contribute to growth through its indirect impact on private research.

2.3 Welfare Properties

Finally, we close this section by describing the SBGP equilibrium welfare. In an SBGP equilibrium that has an initial consumption C_0 and a growth rate of g, welfare is computed as

$$W(C_0, g)^{SBGP} = \int_0^\infty \exp\left(-\delta t\right) \frac{\left(C_0 e^{gt}\right)^{1-\gamma} - 1}{1-\gamma} dt = \frac{1}{1-\gamma} \left(\frac{C_0^{1-\gamma}}{\delta - (1-\gamma)\, g} - \frac{1}{\delta} \right).$$

We will report our results in consumption-equivalent terms. In particular, when two different public policies T_1 and T_2 generate different SBGP equilibrium welfare values as $W(C_0^{T_1}, g^{T_1})$ and $W(C_0^{T_2}, g^{T_2})$, we will report β such that

$$W(\beta C_0^{T_1}, g^{T_1}) = W(C_0^{T_2}, g^{T_2}).$$

In other words, β constitutes the compensating differential in initial consumption that equalizes the welfare of the two proposed policy environments. Therefore, it provides an intuitive measure for evaluating policy tools.

To sum up, let us briefly discuss the sources of inefficiency and what policy can achieve in this model. First, as in standard quality ladder models, there are intertemporal spillovers within each product line. Second, firms simply enjoy the expected duration of monopoly power due to the competition channel of creative destruction. As a result, the private value of innovation differs from the social value of innovation. It is also worth highlighting that, in this model, there could be either over- or underinvestment in R&D. In addition to the standard channels, our model features additional spillovers due to basic research, both within and across industries. Finally, there are additional static distortions due to monopoly power. However, since we are interested primarily in the dynamic inefficiencies associated with innovation and basic research, we will consider the case of a social planner who is still subject to monopoly distortions on the production side.

In the full dynamic framework, all of these inefficiencies will generate room for innovation policy, and our estimated model will govern whether there is over- or underinvestment in the various types of research expenditures in the decentralized equilibrium. It will also provide a framework within which we can evaluate the effects of these innovation policies.

2.4 Social Planner

For the social planner's problem, we choose innovation rates to maximize steady state welfare, while taking static monopoly distortions as given. Furthermore, we assume that all spillovers from both the public and private research sectors are immediately internalized. This allows us to make a number of simplifications. First, both the labor cost and growth impact of applied innovation, be it by incumbents or entrants, become the same, so we need only choose a common rate of applied innovation a for a mass $1 + E$ of applied research labs. Similarly, the same is true for private and academic basic research, and so we need only choose a common rate of basic innovation b. There is still the extensive margin decision for private basic research labs, which we characterize with a threshhold value $\hat{\phi}$ for the random fixed cost.

As seen in equation (20), initial output (and hence consumption) is simply a function of the production labor allocation, which is equal to one minus the research labor allocation. Thus, we can perform the following constrained maximization over the logarithm of welfare

$$\begin{aligned} \max_{a,b,\hat{\phi}} \quad & (1-\gamma)\log(1-R) - \log(\delta - (1-\gamma)g) \\ \text{s.t.} \quad & P = \mathbb{P}[\phi < \hat{\phi}] \\ & \tau_a = (1+E)a \\ & \tau_b = (P+U)(1+p)b \\ & R = (1+E)h_a(a) + (P+U)h_b(b) + P\mathbb{E}[\phi|\phi < \hat{\phi}] + U\bar{\phi} \\ & g = g(\tau_a, \tau_b) \quad \text{(see eq. 21)} \end{aligned} \tag{22}$$

Before moving on to the quantitative analysis, it is useful to provide a discussion of the key assumptions of our model and their implications on the results. This also allows us to better convey the theoretical structure underpinning our quantitative estimates and the associated policy results.

2.5 Modelling Choices and Assumptions

As in other endogenous growth models, innovations in our model occur thanks to the monopoly rents that are protected by patents.[16] Since patent protections are in place, competitors cannot simply copy the frontier technology, and they need to come up with a technology that is superior to the incumbent's technology. This business stealing effect introduces a source of overinvestment in R&D. At the same time, firms are building on the shoulders of giants (i.e., past innovations before time t make the current innovation step size increase by $\bar{q}_i(t)$), introducing a source of underinvestment in R&D. Hence, our choice of Schumpeterian models provides the possibility that the decentralized economy could feature over- or underinvestment in R&D.

The reader should note that we have full business stealing, in the sense that entry completely pushes out the previous incumbent in that product line. In an alternative specification, one could imagine some type of patent sale or licensing arrangement. Such an arrangement would induce the need for payments between incumbents and new entrants. Consequently incumbents would potentially have blocking power and receive a stream of royalties. This might affect the model's efficiency properties as they relate to within industry spillovers, but would likely not change the inefficiencies arising from uninternalized cross-industry spillovers. In our model, we abstract from

[16] To see an explicit treatment of patent policy in this class of models, see Acemoglu and Akcigit (2012), among others.

such possibilities mostly because, to the best of our knowledge, there is no systematic data on such licensing transactions that could discipline this margin.

Another important feature of the model is that creative destruction happens because rival firms are innovating with a technology that makes the previous technology (and its patent) obsolete. Our model therefore ignores the possibility that firms may improve their own products through internal innovation, without stealing someone else's product through external innovation. Introducing internal innovations into this model would have an ambiguous impact on innovation incentives. On the one hand, firms would internalize the intertemporal spillovers that the internal innovation generates, and increase their innovation efforts. On the other hand, this mechanism would reduce the business stealing margin, and therefore reduce investment into research. However, as in the previous discussion, the inefficiency properties associated with the cross-industry spillover should remain the same. Empirically, there is some disagreement in the literature about the relative importance of internal versus external innovations.[17] Hence, while we appreciate the fact that this additional distinction could be interesting to incorporate, we prefer to leave this as an extension for future research.

A key distinction introduced by our model is the split between basic and applied research. Importantly, the model structure has enough flexibility to allow for quantitatively different or similar spillovers within and across industries, depending on the estimated step sizes (η, λ). In addition, since the government is also investing in basic research through public research labs, it could be that, depending on how much money is allocated to them, the decentralized economy over- or under-invests in basic research relative to applied research. Therefore, the efficiency properties become a quantitative question, and our model has the ability to stimulate meaningful policy discussions in this domain.

Finally, in addition to distinguishing between basic and applied research, we also consider whether research is undertaken by the private or the public sector. Universities are a major source of basic research in our model, and an important simplifying assumption is that their only goal is to improve productivity. This could be a strong assumption since universities and policymakers also care about other educational aspects of knowledge creation and knowledge diffusion. While it is difficult to precisely quantify the weight attached by policymakers to each objective, such additional benefits would lead us to under-estimate the optimal allocation of funds to the public research sector. In our model, such additional objectives could be introduced by changing the objective function of the social planner.

3 Data, Measurement, and Empirical Evidence

In this section, we present and discuss the empirical patterns that provide the basis for the structural estimation in Section 4. First, we describe the data sources that allow us to document patterns of research investment and patenting decisions. Second, we show how the research investment patterns of diversified firms allow us to identify cross-industry spillovers of research. We then use patent

[17] For instance see Garcia-Macia, Hsieh, and Klenow (2019) and Akcigit and Kerr (2018).

citation data across public and private inventors to quantify step sizes in basic and applied research, as well as the associated cooldown rate.

3.1 Data and Variable Construction

To test and estimate our model, we rely on a unique combination of datasets on French innovative firms between 2000 to 2006. The final sample is composed of 13,708 firm-year observations, for which we provide further details regarding data organization and descriptive statistics in Appendix A.

R&D Information Information about research investment comes from the annual R&D Survey conducted by the French Ministry of Research. The survey is conducted in annual waves of cross-sectional data, and covers a representative sample of French firms of more than 20 employees investing in R&D. The questionnaire includes extensive information about the financing of R&D. It not only breaks down R&D investment according to the source of the funds, but also provides the allocation to different types of research.

The distinction between basic and applied research is based on survey standards defined by the OECD in the Frascati manual. Basic research investment refers to a "systematic study to gain more comprehensive knowledge or understanding of the subject under study without specific applications in mind." Conversely, applied research is defined as a "systematic study to gain knowledge or understanding to meet a specific, recognized need."[18]

One possible concern is that it is hard to conceptually draw a line between what constitutes basic scientific research and applied scientific research. Indeed, there is a continuous spectrum of scientific activity and "moving from the applied-science end of the spectrum to the basic science end... [,] goals become less clearly defined" (Nelson, 1959). Similarly, Rosenberg (1990) argues that it is challenging to categorize research on the basis of the motives of the person performing it. We address this concern in two ways. First, we note that several seminal papers in the literature (Mansfield, 1980; Mansfield, 1981; Link, 1981; Griliches, 1986) show that the distinction between basic and applied research in this type of data is not only conceptually useful, but also quantitatively important.[19] Second, we also consider the measurement issue from within the context of our model and quantitative results. In particular, we explicitly allow for misclassification of research type in Section 5.5. By doing so, we consider how policymakers could set policies in the presence of various levels of misclassification.

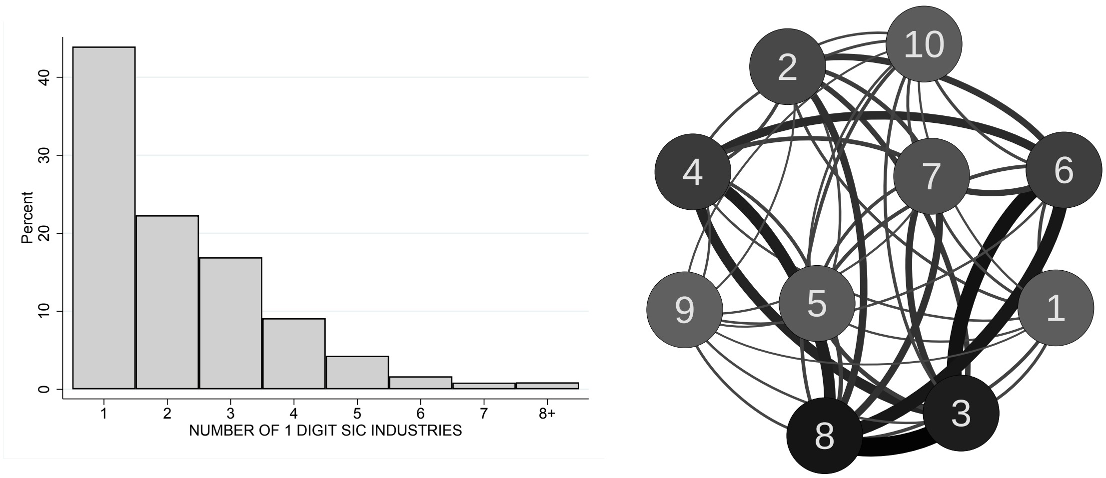
Multi-Industry Activity & Balance-Sheet Information To measure industry presence, we use information from "Enquete Liaisons Financieres" (LIFI) and "Enquete Annuelle des Entreprises" (EAE).

[18]Descriptive statistics are provided in Appendix A. One should note that the definition of corporate basic research does not exclude that it may be directed to fields of current or potential interest, as in the examples provided in the introduction. Importantly, our measurement of research expenditures is based on "intramural" activities and allows us to clearly distinguish research developed within the company from research outsourced to universities or to other units in the group.

[19]These papers typically evaluate the importance of basic and applied research by relating it to production data and estimating its output elasticity or rate of return from an extended Cobb-Douglas production function. The common conclusion from those papers is that basic research seems to command a significant "premium"–i.e., that its contribution to firm productivity is significantly larger compared to that of applied research.

The former allows us to distinguish activities within the same business group, while the latter distinguishes between activities within the same firm and provides us with accounting data. All the data sources are connected through unique firm identifiers allowing us to match them to the R&D data.

Figure 4: Distribution and Links of Firms Across Industries



The figure uses 13,708 firm-year observations from the pooled data for the period 2000-2006. The left panel plots the share of firms as a function of the *number of 1-Digit SIC industries*, which is the number of distinct SIC codes in which a firm is present. The right panel plots the pairwise presence of firms across industries through nodes and lines. Node color is assigned in terms of total number of firms in the industry (lighter to darker). Thickness of lines between industries is assigned in terms of the total number of links between each pair. Activity classification: 1 "Agriculture", 2 "Food and Textile Industries", 3 "Manufacture of chemicals, metals and machinery", 4 "Manufacture of electrical and transport equipment", 5 "Construction and Utilities", 6 "Wholesale and retail trade", 7 "Transport, Communication and Financial Activities", 8 "Professional, scientific and technical activities", 9 "Education and Human Health Activities", 10 "Arts, entertainment and others".

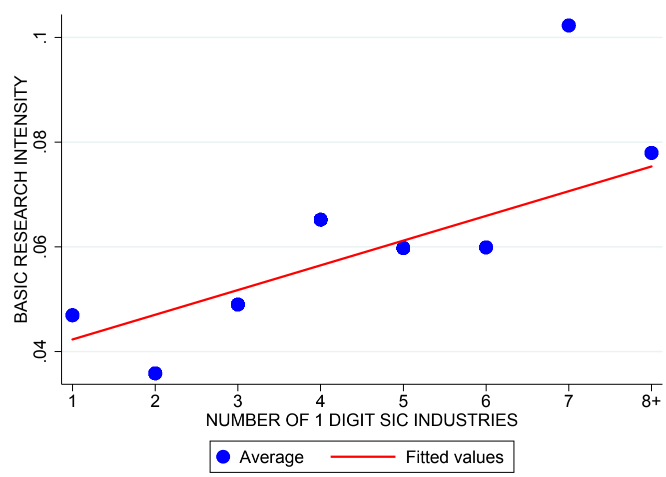
Figure 4 plots the multi-industry presence of firms and the direction of these links. The left panel of Figure 4 plots the share of firms as a function of the number of 1-Digit SIC industries, which is the number of distinct SIC codes in which a firm is present. On average, firms are present in two distinct industries, as defined by 1-digit SIC codes. Although nearly 44% of the firms are operating in only one industry, the remaining firms occupy a large spectrum of industries. Results are very similar when using more disaggregate SIC classifications (up to the 4-digit SIC level) or when changing the definition of an industry link. The right panel of Figure 4 plots the pairwise presence of firms across industries through nodes and lines. Each node color represents the total number of firms in the industry (darker means more firms), while the thickness of the lines between industries represents the total number of links between each industry pair. The figure shows that the most intense industry links flow between manufacturing industries (node 3) and scientific activities (node 8), while pharmaceutical firms (node 9) are less connected to the rest of the innovative firms. One concern is that our measure of multi-industry presence could be driven by the presence of financial units of firms. While some of the links also flow through financial firms included in "Transport, Communication and Financial Activities" (node 7), they do not appear to dominate in any way. In Column (2) of Table 14, Appendix B, we recompute our measures for multi-industry presence and size by excluding financial links and show that our estimate for the cross-industry spillover remains unaffected.

3.2 Investment Across Research Types

We next identify the cross-industry spillovers associated with basic research through the investment choices of multi-industry firms. To do so, we exploit the intuition provided in Nelson (1959): as the range of a firm's products and industries becomes more diversified, its incentive for investing in basic research relative to applied research should increase due to better appropriability of potential knowledge spillovers.

Figure 5 puts together firms' basic research investment and their multi-industry presence. It plots the ratio of basic to applied research according to firms' multi-industry presence. The figure already controls for firm size, measured in terms of log employment, to disentangle it from the multi-industry effect. The figure provides a first visual confirmation that investment in basic research increases as the scope of a firm's activities expands.

Figure 5: Basic Research Intensity Across Industries



The figure plots *Basic Research Intensity* as a function of *Number of 1-Digit SIC Industries*. For each firm-year observation *Basic Research Intensity* is defined as the ratio of total firm investment in basic research divided by total firm investment in applied research. *Average* is the average basic research intensity for firms conditional on the number of their 1-Digit SIC industries and after partialling out for firm size, while the red line represents a linear fit of the firm-year observations.

Table 2 further strengthens the identification of the cross-industry spillover by adding controls for firm and industry heterogeneity. To account for zeros in the the dependent variable, we estimate a Tobit model and report marginal effects.

The baseline specification in column (1) controls for firm size, as measured by log employment. The estimates suggest that a firm's presence in an additional industry is positively correlated with its propensity to invest into more basic research. In terms of magnitude, an additional industry link increases the firm's basic research intensity by three percentage points, on average, corresponding to a 50% increase in the average basic intensity of a single-industry firm.

To better understand whether industry characteristics are the main drivers of our estimated

Table 2: Basic Research and Multi-Market Activity - Industry and Firm Heterogeneity

	(1)	(2)	(3)	(4)
Log # of Industries	0.032***	0.025***	0.024***	0.024***
	(0.006)	(0.008)	(0.008)	(0.008)
Log Employment	0.003**	0.002	0.002	-0.000
	(0.001)	(0.001)	(0.001)	(0.002)
Year & Organization FE	Yes	Yes	Yes	Yes
Industry FE	No	Yes	Yes	Yes
Areas of Research Shares	No	No	Yes	Yes
Patent Stock & No Patent FE	No	No	No	Yes
N	13708	13708	13707	13708

Notes: Pooled data for the period 2000-2006. Estimates are obtained using Tobit models and relate to the marginal effect of the variables at the sample mean. *Basic Research Intensity* is defined as the ratio of total firm investment in basic research divided by total firm investment in applied research. *Log # of Industries* is the number of distinct SIC codes in which a firm is present. *Log Employment* is the total employment of firms. *Share in Software/Biotech/Materials* is the share of R&D investment in the respective areas of research with share of environment research omitted. *Log Patent Stock* measures the number of patents obtained by firms in the pre-sample period (1993-2000). *Industry FE* denote fixed effects for 1-digit SIC activities. *Year FE* denotes year fixed effects, and *Organization FE* denotes whether the firm operates its activity as a conglomerate or as a business group. See the Appendix for the definition of variables. Robust standard errors clustered at the firm level are in parentheses. One star denotes significance at the 10% level; two stars denote significance at the 5% level; and three stars denote significance at the 1% level.

spillover parameter, we proceed in two steps. In Column (2), we first augment the specification relating basic research intensity to multi-industry presence by including a set of industry fixed effects. This specification, therefore, disentangles the spillover effect from industry heterogeneity in research. While the point estimate is slightly lower, the direction and the strength of the correlation remain very similar to the baseline estimates reported in Column (1). A potential concern is that the industry fixed effects might fail to appropriately capture the transversal nature of research. For instance, the multi-industry structure of a firm might be due to its focus on IT research. To address this concern, we exploit information contained in the R&D survey about the amount spent on different areas of research. The survey distinguishes among the following areas: software, biotech, environment, materials, and social sciences. We augment our specification by including controls for areas of research in column (3), and estimates remain very stable.[20]

Finally, in column 4, we try to capture firm-specific heterogeneity in innovative capacity. To do so, we follow an insight provided by Blundell, Griffith, and Reenen (1995). Their study estimates the determinants of innovation activity and tries to overcome the fact that "such things as the different appropriability conditions of research efforts and technological opportunities facing firms" are difficult to measure. The authors argue that the permanent firm-specific capacities for successful innovation should be reflected in the pre-sample history of innovative success. Similar to their study,

[20]We also re-estimated the spillover parameter excluding an outlier industry in terms of basic research intensity that we identify in the Appendix. Estimates were unaffected.

we include the number of patents obtained by firms in the pre-sample period (1993-2000) as a variable that approximates the knowledge stock of the firm at its point of entry into the sample. Again, our estimates remain very similar in economic and statistical terms.

Table 3 tries to empirically address concerns about reverse causality – i.e., a firm's comparative advantage in basic research explaining its expansion into multiple industries. In an ideal setting, we would vary a firm's economic scope in order to observe its impact on research patterns. To get as close as possible to this framework, we follow an instrumental variable approach and exploit an historical event that affected the economic scope of firms.

In 1981, Francois Mitterrand became president of the French Republic and implemented a vast nationalization program across industries. Even before his election, the tradition of French state intervention had resulted in a significant fraction of the economy being under state control. Consistent with Colbertist policies, the state modified the economic scope of firms by merging often unrelated firms into large conglomerates of national champions. Cohen and Bauer (1985) eloquently describe the phenomenon as a "game of monopoly" played by politicians. In 1987, the election of Jacques Chirac on a liberal platform marked the beginning of privatizations, which then continued into the 1990s under the impetus of the European Commission's competition directorate. Using historical data on ownership structures, we define state participation in 1985-1987 as our instrumental variable.

The first-stage estimates reported in columns (1) to (3) of Table 3 confirm that state participation in the 1980s is associated with, on average, 1.5 more industry links for firms between 2000 and 2006. The associated F-test is well above the critical levels related to weak instruments. The second-stage estimates, reported in column (4), suggest that the instrumented estimate is significantly larger than the OLS one.[21] This would suggest that our estimates might be a relatively conservative quantification of spillovers.

The exclusion restriction associated with our identification strategy requires that state participation in the 1980s should not directly affect a firm's research incentives in the year 2000. To strengthen the argument, all of our IV specifications include a measure of continued state participation in the ownership of the firm. Another possible objection is that past ownership by the state also affects the likelihood of accessing subsidies and government contracts. In columns (2) and (5), we include as a control variable the share of public funding obtained by these firms. While public funding of private research is associated with higher basic research, its effect does not seem to confound our IV estimate related to the spillover. In columns (3) and (6), we again exploit the R&D survey to construct a dummy variable for links between private companies and universities. Again the estimates remain very similar and seem to exclude a differential access to the knowledge created by the public university system.

Taken together, the estimates across these specifications confirms the robustness of the correlation between firms' multi-industry presence and their investment in basic research.[22]

[21]The fact that the OLS estimate represents a lower bound to our spillover parameter is consistent with agency theories in financial economics that highlight the difficulty of capital allocation in diversified firms and the associated valuation discount (Jensen and Meckling, 1976; Rajan, Servaes, and Zingales, 2000; Scharfstein and Stein, 2000).

[22]Table 14 in Appendix B further addresses concerns about the measurement of the cross-industry spillover. Column (1)

Table 3: Basic Research and Multi-Market Activity - Instrumental Variables

	1st Stage			2nd Stage		
	(1)	(2)	(3)	(4)	(5)	(6)
State Participation in 1985	.293***	.288***	.264***			
	(.056)	(.057)	(.056)			
Log # of Industries				0.085***	0.079***	0.070**
				(0.018)	(0.021)	(0.028)
Log Employment	.120***	.120***	.117***	-0.007**	-0.006	-0.006
	(.003)	(.004)	(.004)	(0.003)	(0.004)	(0.004)
State Participation	-.057	-.057	-.049	-0.003	-0.005	-0.002
	(.083)	(.083)	(.082)	(0.011)	(0.012)	(0.013)
Public R&D Subsidies		.013*			0.010***	
		(.008)			(0.004)	
Outsourcing to Univ.			.067***			0.019**
			(.012)			(0.010)
Year & Organization FE	Yes	Yes	Yes	Yes	Yes	Yes
Areas of Research Shares	Yes	Yes	Yes	Yes	Yes	Yes
N	13707	13707	13707	13707	13707	13707

Notes: Pooled data for the period 2000-2006. Estimates are obtained using Tobit models and relate to the marginal effect of the variables at the sample mean. *Basic Research Intensity* is defined as the ratio of total firm investment in basic research divided by total firm investment in applied research. *Log # of Industries* is the number of distinct SIC codes in which a firm is present. *State Participation in 1985* is a binary variable equal to 1 if the French state was a shareholder of the firm in 1985. *State Participation* is a binary variable equal to 1 if the French state is currently a shareholder of the firm. *Public R&D Subsidies* is a binary variable equal to 1 if the firm obtained public R&D subsidies. *University Collaboration* is a binary variable equal to 1 if the firm collaborates/outsources some of its R&D to the public system. *Log Employment* is the total employment of firms. *Share in Software/Biotech/Materials* is the share of R&D investment in the respective areas of research. *Industry FE* denote fixed effects for 1-digit SIC activities. *Year FE* denotes year fixed effects, and *Organization FE* denotes whether the firm operates its activity as a conglomerate or as a business group. See the Appendix for the definition of variables. Robust standard errors clustered at the firm level are in parentheses. One star denotes significance at the 10% level; two stars denote significance at the 5% level; and three stars denote significance at the 1% level.

3.3 Citation Patterns Across Research Types

We next use citation information from the NBER patent data to empirically measure the quality step size for basic and applied innovation, as well as their importance for follow-up innovations (Hall, Jaffe, and Trajtenberg, 2001). An empirical issue is how to distinguish between patents derived from basic and from applied research. Trajtenberg, Henderson, and Jaffe (1997) document that patents from the public sector are more basic-research oriented than patents from the corporate sector. Consequently, we use the assignee code to distinguish between patents applied for by corporations from

presents the benchmark specification. In column (2), we recompute multi-industry presence and size by excluding links to banks, insurance companies, and asset management companies. In addition, we explicitly control for the presence of financial activities within the group (either as a subsidiary or as the head of the group). Column (3) recomputes multi-industry presence and size by considering only activities with at least ten employees. We also add two variables in the specification that should capture the possibility of spurious industry presence. The first is a dummy variable on whether the headquarters are foreign. The second is a Herfindahl index related to the concentration of employment within a given industry of the firm's portfolio. Column (4) uses the data on the population of French firms and measures the frequency of each distinct activity pair. We then inversely weight the distinct bilateral links of our innovative firms. All estimates remain robust.

patents applied for by public institutions.[23] The evidence on citation patterns across public- and private-sector patents is organized in Table 4 and explained in Figure 6.

In Panel A of Table 4, we provide a first breakdown of public and private patents according to the number of direct, 1^{st} generation citations that the originating patent received. In the model, the citation distributions for both public and private patents are directly sensitive to the underlying step sizes. The larger the step size, the more citations a patent receives, on average. We find that, on average, a public patent obtains 1.12 more citations than corporate patents.

To estimate the cooldown parameter, we proceed in several steps. First, we again consider whether the originating patent is public or private. Second, for the estimation of the cooldown parameter, instead of computing the 1st generation direct citations (middle panel of Figure 6) received by the original patents, we measure the importance of these 1st generation patents through the number of patent citations they received in turn from 2nd generation patents (bottom panel of Figure 6).[24] We do so for each year since the granting date of the original patent. Panel B reports the mean difference of this measure between patents originated in the public sector and patents originated in the private sector. We find that the data display a pattern whereby owners of patents following/citing a public-sector-originated patent within a few years are of higher quality than those citing private-sector-originated patents. The difference in the quality of follow-up patents oscillates between 0.3 to 0.6 citations in the early years. However, it drops significantly in the 8th year. The results are similar when using the Wilcoxon-Mann-Whitney test. The second row adjusts for the fact that patents from basic research have a broader applicability. Therefore, we use the concordance table developed by Silverman (1996), linking the International Patent Classification (IPC) system to the U.S. Standard Industrial Classification (SIC) system. This concordance enables us to condition citations on being within the same SIC code and qualitatively confirms the previous pattern.[25]

One potential limitation of the above analysis is that the propensity to patent research could be different between the public and private sector and might lead us to mismeasure the role of basic research. Unfortunately, such a differential pattern is hard to quantify with the current patent information and would require new micro-data to be collected. Hopefully our paper will spur the collection of such data and allow a better understanding of the production function of the public sector.

[23] The use of US patent data was linked to the availability of a long time horizon of publicly available data on patents granted, depositor classification, and the associated citations. The analysis of our final dataset will focus exclusively on French patenters, but the construction of the different variables will use information from the entire dataset. While our proxy is simple to measure in the data, it potentially misclassifies the contribution of private basic patents. However, given that our interest lies in the relative difference between those two groups of patents across time, time-invariant errors in the classification should not impact our conclusions. Appendix C provides additional robustness checks for the estimates on the cooldown rate of patents originating from basic and applied research.

[24] All citation measures are based on cumulative 10-year citation horizons to avoid truncation effects.

[25] It is likely that the use of concordance tables introduces some degree of measurement error, and explains why the citation patterns is more noisy relative to the previous row.

Figure 6: Construction of Citation Information

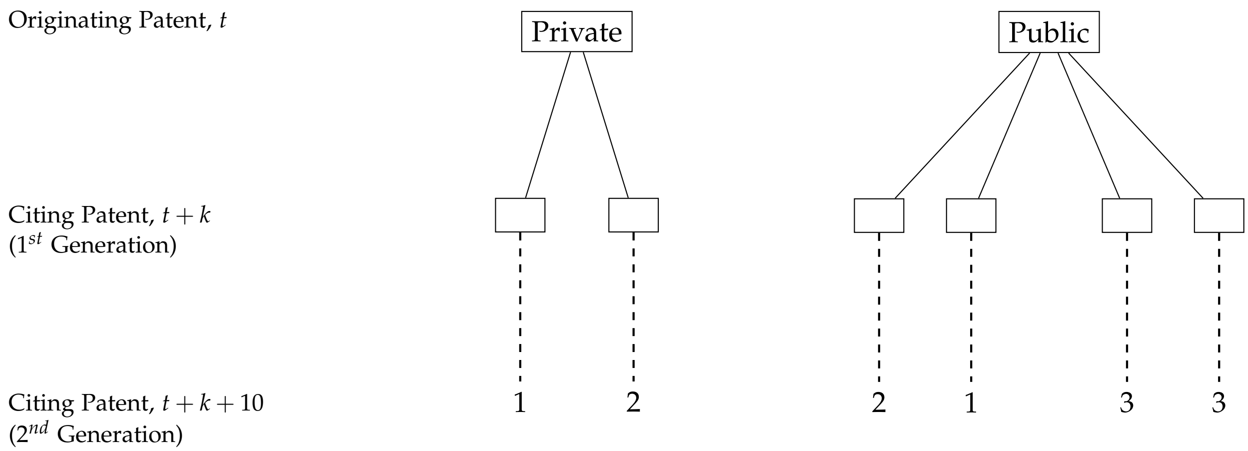


Table 4: Patent Citations Across Public and Private Patents

					Panel A: 1^{st} Generation Patents					
			Mean	25th	Median	75th	St. Dev.	Min	Max	N
Private Citations			5.88	1.00	4.00	7.00	8.01	0.00	179.00	28247.00
Public Citations			7.01	2.00	4.00	8.00	11.86	0.00	351.00	2806.00
					Panel B: 2^{nd} Generation Patents					
Age Cohort	1	2	3	4	5	6	7	8	9	10
Mean Difference	.3**	.3**	.62***	.28**	.41**	.23	.71***	.08	.39	.14
	(0.15)	(0.15)	(0.17)	(0.14)	(0.18)	(0.17)	(0.25)	(0.16)	(0.25)	(0.24)
Adjusted	.3**	.23*	.55***	.26*	.46**	.18	.58**	.06	.47*	.22
	(0.15)	(0.14)	(0.17)	(0.17)	(0.17)	(0.17)	(0.24)	(0.15)	(0.26)	(0.28)

Notes: Citation patterns of patents granted by the USPTO to French private and public depositors. Panel A computes direct 1st generation citations across public and private patents. Panel B reports the average difference in citations of 2nd generation patents for the period 1975-1990. Adjusted citations are conditioned on being within the same 1-digit industry of use, as indicated by the concordance table in Silverman (2004). Two sample t-tests with unequal variances were used. One star denotes significance at the 10% level; two stars denote significance at the 5% level; and three stars denote significance at the 1% level.

4 Quantitative Analysis

4.1 Estimation

In this section, we describe the estimation strategy used. We assume that the idiosyncratic fixed-cost component of basic research is drawn from a lognormal distribution with $\log(\phi) \sim \mathcal{N}(\bar{\phi}, \sigma^2)$. As a result, the set of parameters of the model is

$$\theta = \{\delta, \gamma, \varepsilon, p, \eta, \lambda, E, U, \nu_a, \nu_b, \xi_a, \xi_b, \kappa, \bar{\phi}, \sigma, \zeta, \varsigma\} \in \Theta.$$

In our dataset, for each firm f and each time period t, we have a vector of N observables from the actual data $\mathbf{y}_{ft} \equiv \left[y^1_{ft} \; . \; . \; . \; y^N_{ft}\right]'_{N\times 1}$ that includes the number of industries in which the firm is present, sales, profits, and labor costs. Let the entire dataset be denoted by $\mathbf{y}$.

We use the simulated method of moments (SMM) for the estimation.[26] Define $\Lambda(\mathbf{y})$ and $\Lambda(\theta)$ to be, respectively, the vectors of real data moments (generated from $\mathbf{y}$) and equilibrium model moments (generated for some vector of parameters θ). Since certain moments require knowledge of the joint distribution of firms over the number of products and industries (m, n) and the portfolio of product qualities $\mathbf{q}$, which has no apparent analytic form, we simulate a large panel of firms to calculate $\Lambda(\theta)$ to a high degree of accuracy.[27]

Our proposed estimator minimizes a quadratic form of the difference between these two vectors

$$\hat{\theta} = \arg\min_{\theta \in \Theta} [\Lambda(\theta) - \Lambda(\mathbf{y})] \cdot W \cdot [\Lambda(\theta) - \Lambda(\mathbf{y})], \tag{23}$$

where W is the weighting matrix. We use a diagonal weighting matrix with entries equal to the inverse square of the data moment value, or, in notational terms, $W_{ii} = 1/\Lambda_i(\mathbf{y})^2$ and $W_{ij} = 0$ for $i \neq j$. In our estimation, we use 30 moments, which we denote as $\Lambda(1) - \Lambda(30)$. We pick moments that are most informative for identification of the unique features of our model.

Status Quo Policies We also take into account that, during the period we consider, there was existing government support for R&D activities in France. In our dataset, 10% of corporate R&D is, on average, publicly funded. Therefore, in our estimation, we introduce a uniform subsidy to the total R&D spending of the firm $\psi_a = \psi_b = 0.10$. The government has a balanced budget every period, so that the sum of total subsidies (S) and public research funding (R) must be equal to tax revenues; that is,

$$T = S + R = \psi_a h_a(a) + \psi_b \sum_{m=1}^{M} \mu_m P_m \left[h_b(b_m) + \bar{\phi}_m\right] + U\left[h_b(u) + \bar{\phi}\right],$$

where T is a lump-sum tax on consumers. In France, during 2000-2006, the fraction of GDP devoted to public research labs and academic institutions was approximately 0.5%. Therefore, we pick R/Z, which is the share of GDP devoted to public basic research, to be 0.5%.

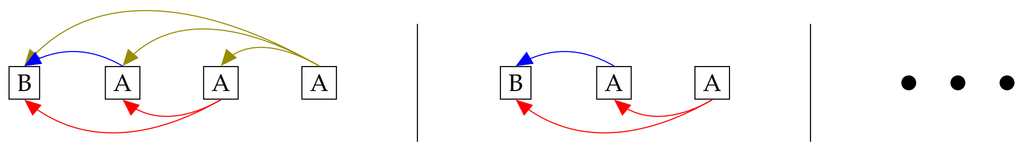
4.2 Identification

Ultimately, the parameters of the model are identified jointly by their optimality with respect to the estimation objective function defined above. Nonetheless, there is a clear sense in which certain parameters are identified mainly through certain moments. In this section, we provide an account of such relations with special focus on those parameters that are either novel to our model or are particularly important for the ensuing policy exercises. Additionally, we provide the full Jacobian matrix of the moment vector Λ with respect to the parameter vector θ. This provides precise numerical values for the direction and magnitude of moment-parameter dependence in the neighborhood of our estimated parameter values.

[26] See Bloom (2009) and Lentz and Mortensen (2008) for further description and usage information on SMM. Appendix E provides additional details about the computer algorithm.

[27] For our results, we simulate 32K firms with a burn-in time of 100 years and 100 time steps per year.

Figure 7: Citation Model Diagram



Basic innovations are represented by "B", while applied innovations are represented by "A". Each arrow represents the possibility of a citation. An actual citation will occur with probability ηx for basic innovations and λx for applied innovations.

4.2.1 Step Sizes: Citation Model

The innovation step sizes for basic and applied research are two of the most critical parameters in the model, as they fix the scale of innovation sizes against which input costs are weighed. Though these values are important in determining the overall growth rate, profitability of firms, and the firm-size distribution, the distribution of patent citations provides the most direct evidence. In the model, the citation distributions for both public and private patents are directly sensitive to the underlying step sizes. The larger the step size, the more citations a patent receives, on average.

The distinction between basic and applied research has a very natural interpretation in this setting. We interpret each basic innovation as starting a new line of research that is cited by subsequent applied innovations that build off of it. When a subsequent basic innovation is generated, the original basic patent receives no more citations, and the new line of research prevails. Similarly, applied innovations also build off of one another, and we allow them to cite prior patents both from basic and applied research. We model the probability of being cited by subsequent innovations as some parameter x times the step size of the innovation. This encapsulates the notion that patents with a high immediate impact on productivity are also generally more highly cited.

Recall that the two step sizes in our model are η and λ, and that we have applied and basic innovation rates τ_b and τ_a. We show in Appendix A that under the dynamics described above, the total number of citations a patent receives will follow a geometric distribution. In particular, the probability of a patent with step size s receiving n citations will be

$$p_s(n) = (1 - c_s)c_s^n \quad \text{where} \quad c_s = \frac{\tau_a x s}{\tau_a x s + \tau_b}.$$

Additionally, we know that basic innovations always have step size η, while applied innovations have step size η when they land on "hot" product lines (with probability α) and step size λ otherwise. Thus, we find that

$$\begin{aligned} p_b(n) &= p_\eta(n) \\ p_a(n) &= \alpha p_\eta(n) + (1-\alpha)p_\lambda(n). \end{aligned}$$

This structure has the advantage that it not only generates average citation rates and counts for basic and applied research, but also generates an entire predicted distribution of citation counts across patents. Importantly, these distributions are directly sensitive to the underlying step sizes. The larger the step size, the more citations a patent receives, on average. Since the scale parameter x is essentially setting units, this soaks up one degree of freedom. But the remaining difference between basic and applied citations is informative about the difference between the respective step sizes. Meanwhile, τ_b and τ_a are constrained through equilibrium by the firm's dynamics moments, while the absolute step size is constrained by the aggregate growth rate.

While we don't observe patents as being basic or applied directly in the data, we can proxy these differences through the public/private origin of the assignee. Our model yields predictions for mean and root-mean-squared (RMS) citations for both public and private patents separately. We use these four moments in the main estimation. Even though they are not targeted directly, we also find that the distributions of patent citations for both private and public patents match those seen in the data quite well. As seen in Figures 8 and 9, this close match gives us confidence that the way in which we model citations is, indeed, appropriate for this setting.

FIGURE 8: PRIVATE CITATION DISTRIBUTIONS

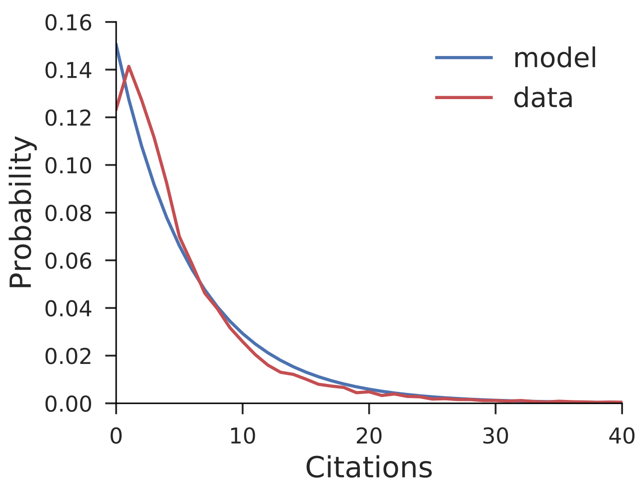
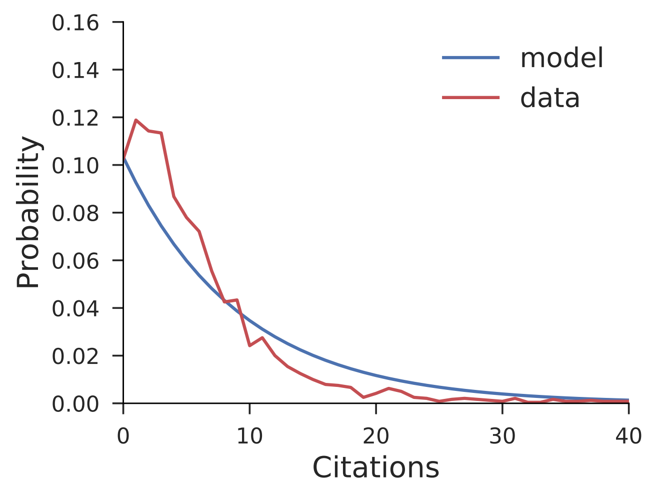


FIGURE 9: PUBLIC CITATION DISTRIBUTIONS



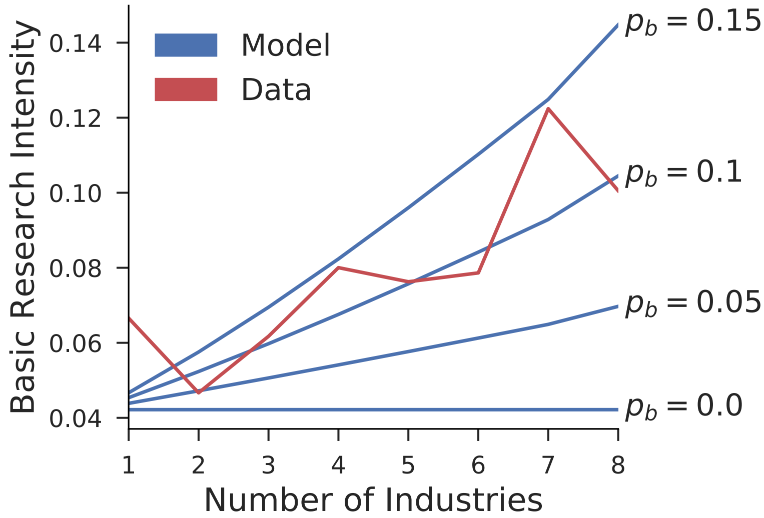
4.2.2 Across-industry Spillovers

We define basic research intensity as the ratio of spending on basic research to spending on applied research. Since the effect of multi-industry presence on this quantity is of critical importance to our model, we have one moment for each $\hat{m} \in \{1, \ldots, M\}$, the number of industries a firm is observed to be operating in. Note that firms in the model might be "present" in an industry but currently have no products there, meaning the observed $\hat{m}$ might be different from the underlying m. Given a set of parameters and an equilibrium of the model, this moment's value for a given $\hat{m}$ is

$$\Lambda(1-8) = \mathbb{E}_m \left[\frac{P_m(h_b(b_m) + \bar{\phi}_m)}{h_a(a_m)} \,\middle|\, \hat{m} \right].$$

In our estimation, we use $M = 10$. However, in the data, there are only a handful of firms with $\hat{m} > 8$, so we have one moment for each $\hat{m} \in \{1, \ldots, 7\}$ and a final moment that is averaged over

Figure 10: Basic Research Intensity vs Spillover Parameter p



$\hat{m} \in \{8, 9, 10\}$. The way in which this moment increases with $\hat{m}$ identifies the cross-industry spillover parameter p in our model. Additionally, the overall level provides us with some identification power for the basic research cost parameters (ξ_b, ν_b).

Figure 10 plots the empirical (red line) and model counterparts for these moments for different spillover parameter $p = \{0.00, 0.05, 0.10, 0.15\}$. The figure makes it clear that our cross-industry spillover parameter p is identified by matching the basic research intensity of multi-industry firms.

The slope of the analogous graph for the extensive margin of basic research (P_m) is also sensitive to the magnitude of cross-industry spillovers. Thus, we use the share of positive basic research spending by each $\hat{m}$ to provide additional information on p. The overall level of positive basic research is also directly informative about the mean $\bar{\phi}$ and variance σ^2 of the fixed-cost component of basic research. Analytically, the probability that a firm with a certain industry presence $\hat{m}$ undertakes basic research is simply the probability that the idiosyncratic fixed-cost draw is less than the cutoff for that value of $\hat{m}$

$$\Lambda(9 - 16) = \mathbb{E}_m \left[P_m | \hat{m} \right].$$

Though not plotted here, one can see in the Jacobian matrix in Table 16 that the average fixed-cost parameter $\bar{\phi}$ strongly influences the level of this moment for all values of $\hat{m}$. Furthermore, the fixed-cost variance parameter σ induces a tilt in these moments across different $\hat{m}$ values. That is to say, increasing σ raises the probability of performing basic research for low $\hat{m}$ firms and lowers it for high $\hat{m}$ firms. This is consistent with the intuition of the model and with the properties of the lognormal distribution.

4.2.3 Within-industry Spillovers

As discussed in Section 3.3, we use patent citation data to quantify the within-industry spillovers associated with basic research. The model predicts that innovations that build off of previous basic research should have, on average, a larger step size. In this setting, innovations with a larger step size also see more citations. Thus, patents that cite basic innovations should themselves have more citations. We empirically confirmed that the magnitude of this effect diminishes with the age of the patent due to product line cooldown ζ.

Consider a public innovation in the model: this will generate a public patent and make a particular private product line hot for some time. During this time, any subsequent (citing) innovations, whether basic or applied, will yield a basic innovation step size η, just as all public patents do. This step size will determine how many citations these citing patents garner down the line. Then, at some point in time, the product line will cool down and again begin yielding step sizes determined by the research type, either λ or η. Thus, the average cooldown time is also the average time after which a public innovation is indistinguishable from a private innovation, which is given by

$$\Lambda(24) = \frac{1}{\zeta}\left(\frac{\tau_a}{\tau}\right).$$

The first term arises due to the standard properties of a Poisson process with rate ζ, while the second term arises from the fact that another basic innovation may hit the product line before cooldown ocurrs, in which case the original line would receive no further citations. This yields direct information on the value of the cooldown rate ζ, as seen in the Jacobian matrix.

4.2.4 Industry Expansion

We track two moments relating to the distribution of $\hat{m}$, the mean and mean squared. They are given by

$$\Lambda(17) = \mathbf{E}_{\hat{m}}\left[\hat{m}\right] \qquad \text{and} \qquad \Lambda(18) = \mathbf{E}_{\hat{m}}\left[\hat{m}^2\right].$$

Looking at the Jacobian matrix, one can see that the mean m value and the mass of potential entrants are closely linked, as successful entrants start with the lowest working knowledge value $m = 1$. Similarly, the mean of m^2 is strongly influenced by the entrant buyout probability parameter ς. Intuitively, when entrants are more likely to be bought out by incumbents, which results in industry expansion with probability z, this will concentrate industry expertise in existing firms, thus increasing the dispersion of m.

Empirically, we match the distribution of firms across industry classes, as shown previously in the left panel of Figure 4.[28]

4.2.5 Remaining Moments and Parameters

The remaining moments used in the estimation of the model include firm-level profitability, employment growth, age, and total research intensity. At the aggregate level, we use the growth rate of the French economy during the sample period. In Appendix F, we provide a detailed discussion of the mapping between these moments and the parameters of the model.

[28] As discussed in Section 3.2, there are both costs and benefits to multi-industry presence. In our setting, a broad economic scope allows firms to better internalize spillovers from research. At the same time, the corporate finance literature has highlighted the difficulty of capital allocation in diversified firms. Thus, the empirical distribution that we target reflects the net trade-off between these forces.

4.3 Estimation Results

Table 5 reports the values of the estimated structural parameters. The estimated values of the discount rate and CRRA utility parameters are within their standard macro ranges. The elasticity of substitution parameter generates 17%$(= 1/\varepsilon)$ gross profits, resulting in 7.9% net profits after subtracting R&D expenses as a share of sales.

TABLE 5: PARAMETER ESTIMATES

#	Description	Sym	Value	#	Description	Sym	Value
1.	Discount Rate	δ	0.039	10.	Applied Cost Curvature	ν_a	1.367
2.	CRRA Utility Parameter	γ	3.020	11.	Basic Cost Curvature	ν_b	1.539
3.	Elasticity of Substitution	ε	5.863	12.	Applied Cost Scale	ξ_a	1.230
4.	Cross-industry Spillover	p	0.116	13.	Basic Cost Scale	ξ_b	5.445
5.	Buyout Rate	ς	0.456	14.	Exogenous Exit Rate	κ	0.006
6.	Basic Step Size	η	0.079	15.	Basic Fixed Mean	$\bar{\phi}$	-4.759
7.	Applied Step Size	λ	0.050	16.	Basic Fixed Std. Dev.	σ	0.329
8.	Mass of Entrants	E	0.495	17.	Product Cooldown Rate	ζ	0.117
9.	Mass of Academic Labs	U	0.493	18.	Citation Rate	x	2.984

One of the most important parameters of our model is the cross-industry spillover parameter $p = 0.12$, which measures the probability that a basic innovation will have any additional immediate applications (it may have many). This estimate affects the extent to which basic innovations contribute to cross-sectional growth. In equilibrium, firms operate in two out of ten industries on average. Therefore, any given spillover is not embodied with probability 89%$(= 8/9)$. Given that the probability of having a spillover is 12%, the probability of having a disembodied spillover is 10%$(= 0.12 * 0.89)$.

The estimated innovation size of basic research is $\eta = 7.9\%$, and the innovation size of each new applied innovation is $\lambda = 5.0\%$. This implies that basic research (hot product lines) makes applied innovation 58%$(= 7.9/5.0 - 1)$ more productive.

Additionally, each basic innovation has a within-industry spillover. The cooldown rate of hot product lines is estimated to be $\zeta = 0.12$, which indicates that a basic innovation affects the subsequent innovations in the same product line for almost 8.5$(= 1/0.12)$ years, on average.

The elasticity of applied innovation counts with respect to the research dollars spent is estimated to be 0.73 $(= 1/\nu_a)$, and, similarly, the elasticity of basic innovation with respect to the basic research investment is 0.65 $(= 1/\nu_b)$. These values are close to the elasticity estimates in the literature, which typically find a value of around 0.5 (Griliches, 1990; Pakes and Griliches, 1984; Kortum, 1992; Kortum, 1993).

4.4 Goodness of Fit

Table 6 contains the moments from the actual data and our estimated model.

The results indicate that the model performs very well in generating firm and industry dynamics

TABLE 6: MOMENTS USED IN ESTIMATION

#	Description	Model	Data	#	Description	Model	Data
Λ(1-8)	Basic Research Extensive	See Figure 12		Λ(23)	R&D/Labor	0.276	0.260
Λ(9-16)	Basic Research Intensive	See Figure 11		Λ(24)	Employment Growth	0.103	0.103
Λ(17)	Mean Industries	2.264	2.203	Λ(25)	Aggregate Growth	0.013	0.015
Λ(18)	Mean Square Industries	7.535	6.976	Λ(26)	Spillover Differential	8.344	8.000
Λ(19)	Return on Sales	0.033	0.033	Λ(27)	Public Citations Mean	8.682	7.013
Λ(20)	Exit Rate	0.092	0.092	Λ(28)	Public Citations RMS	12.63	14.20
Λ(21)	Age, Small Firms	11.77	15.00	Λ(29)	Private Citations Mean	5.747	5.885
Λ(22)	Age, Large Firms	19.22	24.87	Λ(30)	Private Citations RMS	8.581	9.154

similar to those in the data. Consistent with our data, a significant fraction of innovating firms invest in basic research: in our model, in particular, 29% of firms invest in basic research, while, in the data, 27% invest. We also capture the positive relationship between the extensive margin of basic research and multi-industry presence, as evidenced in Table 6 and Figures 11 and 12.

FIGURE 11: FRACTION POSITIVE BASIC BY # INDUSTRIES

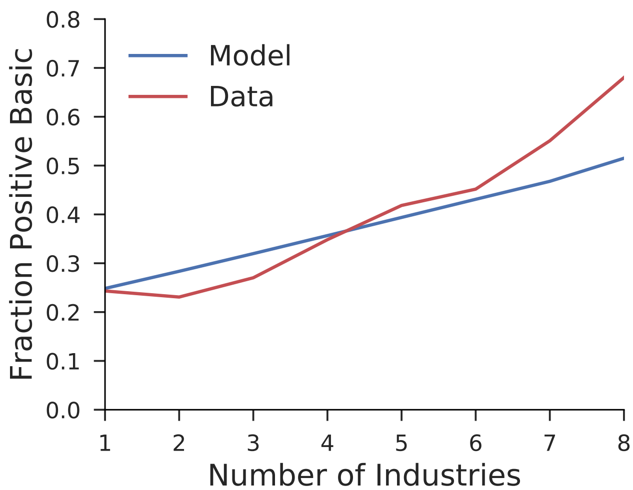
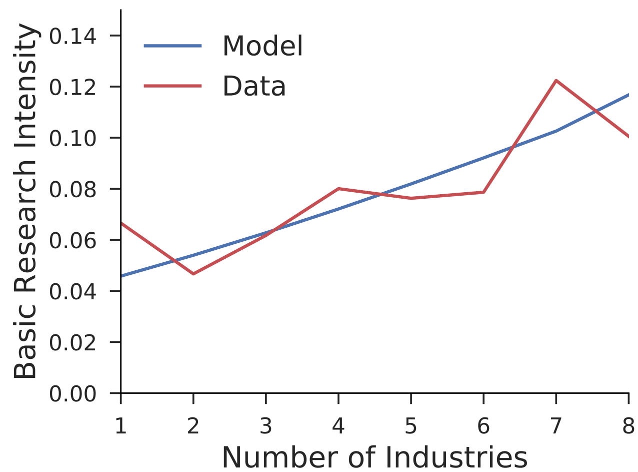


FIGURE 12: BASIC RESEARCH INTENSITY BY # INDUSTRIES



The positive correlation between a firm's multi-industry presence and its basic research intensity was one of the major predictions of our model. As explained previously, multi-industry presence plays an important role in increasing basic research incentives, by allowing a greater potential to internalize the positive spillovers from basic research. In our reduced-form analysis, we confirmed the significant and positive correlation between multi-industry presence and basic research intensity. This has been the key moment to identify the cross-industry spillover parameter. Our model successfully generates this positive correlation.

In the data, firms operate, on average, in 2.2 industries, and the same is roughly true in the model. Furthermore, we find the mean squared in the model to be 7.5, compared to 7.0 observed in the data.

In addition, the mean profitability is 3.2% in the model and in the data. The prime determinants of profitability are the step sizes for basic and applied innovation. However, these also affect the investment levels for both types of research, since this increases the return to successful innovation. Therefore, the step-size parameters are set to be a compromise between hitting the profitability moment and the research investment and growth moments.

All of these components of the economy determine the aggregate growth rate. Our model matches the observed growth rate closely. Our model economy grows at a rate of 1.3%, while the French economy grew at an average rate of 1.5% during the period studied (2000-2006).[29]

5 Policy Analysis

In this section we first study efficiency properties of the decentralized equilibrium in Section 5.1, and the social planner solution in Section 5.2. We then turn to various policies that could address these inefficiencies. Section 5.3 considers a uniform research subsidy that currently exists in many OECD economies. Given our distinction between basic and applied research, we propose in Section 5.4 a hypothetical policy that subsidizes the two types of research differently. This could potentially generate a measurement problem since firms would have an incentive to misreport the type of research they undertake. In Section 5.5, we explore how the scope for misclassification affects optimal policies and welfare. Section 5.6 focuses on the financing and role of the public sector in economic growth. Finally, in Section 5.7, we discuss the robustness of our policy conclusions to extensions of the model.

5.1 Status Quo: Baseline Economy

Table 7 provides equilibrium values for some of the important variables in the model:

Table 7: Decentralized Economy: Endogenous Variables (in percentages)

ψ	R/Z	τ_a	τ_b^e	τ_u	L_p	L_b	L_u	L_e	L_a	α	g	β
subsidy rate	public basic in GDP	applied innovation	private basic innovation	public basic innovation	production labor	labor in private basic	labor in public basic	entrant labor	labor in private applied	share of hot product lines	growth rate	welfare
10	0.5	14.7	0.60	0.13	85.6	0.55	0.52	4.4	8.9	7.2	1.35	100.0

In this table, τ_a denotes the aggregate rate of applied innovation by incumbents and entrants, whereas τ_b^e denotes basic innovation (embodied–i.e., internalized) by private firms, and τ_u denotes basic innovation done by academic research labs. The next five columns report the labor allocations into production, private basic, public basic, entry, and applied research. The remaining columns report the fraction of hot product lines α, the growth rate g, and the welfare in consumption-equivalent terms β.

In our benchmark economy, 85.6% of labor is used for production, and 14.4% for innovation activities. Among researchers, roughly 7% are engaged in basic research activities. Note that this composition within innovation activities will be the central focus of the policy analysis, as uninter-

[29]In Appendix we also discuss the model's prediction about moments that were not directly targeted. These include the correlation between firm profitability and basic research intensity, the correlation between multi-industry presence and firm size, and the skewness of the firm size distribution.

nalized (potential) spillovers are one of the main sources of inefficiency. In order to study the welfare properties of this economy, we normalize the benchmark welfare to $\beta = 1.00$ and compare it to the social planner's optimum, which we will analyze next.

5.2 Quantifying the Social Planner's Optimum

Since our focus is on innovation policies, we consider a social planner who controls the R&D production in the economy while being subject to the same monopoly distortions on the production side (hence, we do not consider a production subsidy in our policy analysis either). Table 8 summarizes these results.

Table 8: Social Planner's Optimum (in percentages)

Policy	ψ_a	ψ_b	R/Z	τ_a	τ_b^e	τ_u	L_p	L_b	L_u	L_e	L_a	α	g	β
Baseline	10.0	10.0	0.5	14.7	0.6	0.13	85.6	0.6	0.52	4.4	8.9	7.2	1.35	100.00
Soc Plan	**-**	**-**	**-**	**12.0**	**3.8**	**1.89**	**83.1**	**4.5**	**2.22**	**3.4**	**6.8**	**35.4**	**1.88**	**104.61**

See Table 7 for variable definitions.

One striking feature of the solution to the social planner's problem under both scenarios is that the fraction of labor devoted to research activities is not substantially greater than in the decentralized equilibrium. In particular, the total labor allocated to production activities is 85.6% in the decentralized economy, while it is 83.1% when set by the social planner. Hence, there is a slight overinvestment in production labor relative to research labor, but this misallocation is quantitatively quite small.

Indeed, the dominant misallocation here is not between production and research, as is common in this class of models, but among the various types of research activities – in this case, applied and basic innovation. In the decentralized economy, only 1.07% of the total labor force is devoted to basic research, whereas in the social planner's economy, this number rises to 6.7%. In other words, the social planner devotes 40% of research labor to basic research, whereas this fraction is only 7% in the decentralized economy. This happens on both the intensive and the extensive margins of basic research. In fact, the planner finds it optimal to employ nearly all private research labs, regardless of their fixed-cost draw.

Another interesting and important finding is that, in the case of applied innovation, there is actually an *overinvestment* in the baseline economy. The applied research labor utilization (including entrants) is 13.3% in the decentralized case. This figure drops to 10.2% in the social planner's solution. This is in spite of the fact that the fraction of hot product lines rises from 7% to 35%, meaning that the average step size of an applied innovation rises by almost a third.

The net result of the above changes is that growth rises from 1.35% to 1.88%. Overall, the decentralized economy's welfare corresponds to a decrease of 4.6% in consumption-equivalent terms from the social planner's optimum. The following policy experiments will try to bridge this gap.

5.3 Uniform Private Research Subsidy

Consider the case in which the government subsidizes a fraction ψ of each firm's total research investment. Note that such a policy subsidizes both basic and applied research, similar to the current R&D subsidy policy in the U.S. and around the world. Funding of academic research is kept unchanged. Table 9 summarizes the results of the optimal uniform subsidy rate.

Table 9: Uniform Research Subsidy (in percentages)

Policy	ψ_a	ψ_b	R/Z	τ_a	τ_b^e	τ_u	L_p	L_b	L_u	L_e	L_a	α	g	β
Baseline	10.0	10.0	0.5	14.7	0.6	0.13	85.6	0.6	0.52	4.4	8.9	7.2	1.35	100.00
Soc Plan	-	-	-	12.0	3.8	1.89	83.1	4.5	2.22	3.4	6.8	35.4	1.88	104.61
Uniform	**29.7**	**29.7**	**0.5**	**16.7**	**1.5**	**0.10**	**82.1**	**1.5**	**0.49**	**5.3**	**10.7**	**13.4**	**1.68**	**100.64**

See Table 7 for variable definitions.

Our analysis of the baseline economy and the planner's economy documented a slight underinvestment in research overall and a large misallocation between the different types of research. A uniform subsidy is, therefore, ill-suited to address these issues, as it cannot directly affect the allocation between research types, and any attempt to subsidize basic research will only worsen the overinvestment in applied research.

Under this policy, we allocate a larger fraction of the labor force to research relative to the social planner's economy. Overall, the researcher's share goes up to 18% from 14%. This new level is quite close to the corresponding level of 17% seen in the social planner's optimum, but the allocation of research labor between basic and applied research is still suboptimal. As a result, the economy grows at a lower rate (1.7%) than the social planner's economy grows (1.9%). The welfare gain from this policy is 0.6 percentage points, which is only 13%$(= 0.6/4.6)$ as large as the gains associated with moving to the social planner's optimum.

Although the underinvestment in basic research is sizable, the uniform policy partially makes up for this, at the cost of worsening the overinvestment in applied research. The main lesson to be drawn from this is that when considering a uniform research subsidy, one should take into account the negative welfare effects associated with oversubsidization of applied research. Finding a feasible method to differentiate between basic and applied research is essential to better innovation policies.

5.4 Type-Dependent Research Subsidy

Now consider the case in which the policymaker sets different subsidy rates for the different types of private research efforts. Let ψ_a and ψ_b denote the applied research and basic research subsidy rates, respectively. The share of GDP allocated to public research (R/Z) is kept constant by the policymaker. Table 10 reports the optimal subsidy rates and resulting equilibrium variables.

Since the underinvestment is mainly in basic research, the optimal type-dependent subsidy dictates a much larger subsidy rate for it – namely, $\psi_b = 49\%$ and $\psi_a = 11\%$. Here, the major component of policy is a more than fivefold increase in the subsidy rate for basic research, whereas the subsidy

Table 10: Type-Dependent Research Subsidy (in percentages)

Policy	ψ_a	ψ_b	R/Z	τ_a	τ_b^e	τ_u	L_p	L_b	L_u	L_e	L_a	α	g	β
Baseline	10.0	10.0	0.5	14.7	0.6	0.13	85.6	0.6	0.52	4.4	8.9	7.2	1.35	100.00
Soc Plan	-	-	-	12.0	3.8	1.89	83.1	4.5	2.22	3.4	6.8	35.4	1.88	104.61
Uniform	29.7	29.7	0.5	16.7	1.5	0.10	82.1	1.5	0.49	5.3	10.7	13.4	1.68	100.64
Targeted	**11.1**	**48.6**	**0.5**	**12.7**	**4.4**	**0.11**	**83.5**	**5.1**	**0.50**	**3.5**	**7.3**	**29.9**	**1.71**	**102.81**

See Table 7 for variable definitions.

rate for applied innovation is virtually unchanged.

The large value for the basic research subsidy is straightforward to understand. There are spillovers associated with basic innovation that are not internalized by firms. By subsidizing this type of innovation, we can mitigate this effect. Contrary to the results seen in the uniform policy case, this policy can achieve a large fraction of the welfare gains seen in the social planner's optimum: 61% (= 2.8/4.6). Still, there is a good deal of room left for improvement, and in the next section, we investigate the sources of these remaining inefficiencies.

5.5 Misclassification of Research Type

The previous discussion highlights that the most promising policy tool to increase welfare is based on a type-dependent subsidy. One concern, however, is that it is not always clear whether or not a specific undertaking should be considered basic research. This might lead firms to misclassify research investment in order to exploit the differential subsidy rates. In this section, we quantify how optimal policies and welfare depend on the degree of misclassification of research.

In this setting, some portion of applied research spending might be misclassified as basic research and, therefore, obtain a more favorable subsidy rate. We can capture this theoretically by assuming that some fraction $z \in [0,1]$ of applied research is misclassified as basic research. Letting v_a and v_b stand for the total present value associated with successful innovation of the respective type, this results in the modified product-line level optimization

$$\Pi = \max_{x_a, x_b} x_a v_a + x_b v_b - (1 - [(1-z)s_a + z s_b])\, \tilde{w} c_a(x_a) - (1 - s_b)\tilde{w} c_b(x_b) \tag{24}$$

Thus, setting real subsidy rates s_a and s_b in this modified regime (characterized by the level of misclassification z) is equivalent to setting subsidy rates $\tilde{s}_a$ and $\tilde{s}_b$ in the regime without misclassification ($z = 0$), where these effective rates are given by

$$\tilde{s}_a = (1-z)s_a + z s_b \tag{25}$$

$$\tilde{s}_b = s_b. \tag{26}$$

Given an optimal type-dependent subsidy scheme $\hat{s}_a$ and $\hat{s}_b$ from the case with no misclassification of research, we can exactly implement this by setting $\tilde{s}_a = \hat{s}_a$ and $\tilde{s}_b = \hat{s}_b$, which, in terms of real subsidy rates means

$$s_a = \frac{\hat{s}_a - z\hat{s}_b}{1-z} \quad \text{and} \quad s_b = \hat{s}_b \tag{27}$$

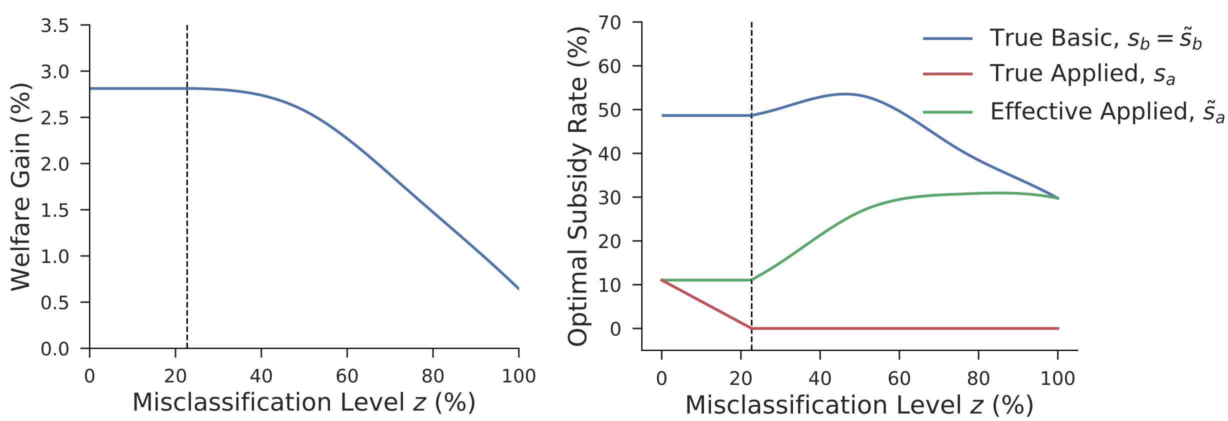
As this expression makes clear, in the region where the desired s_a and s_b are both positive, we can replicate the optimal policy. However, if we are constrained to set positive subsidy rates, the full optimum cannot be implemented. This will be the case when the misclassification parameter exceeds a certain threshold–i.e.,

$$z > z^* \equiv \frac{\hat{s}_a}{\hat{s}_b}. \tag{28}$$

The optimal policies are illustrated in the left panel of Figure 13, where the red line indicates the true applied subsidy rate (s_a); the green line is the effective applied subsidy rate ($\tilde{s}_a$); and the blue line is the true basic subsidy rate that also corresponds to the effective one ($s_b = \tilde{s}_b$). The dashed vertical line represents the threshold at z^*. When the scope for misclassification is small (i.e., $z < z^*$), we can implement the optimal policy by setting the basic subsidy to the exact targeted rate ($s_b = \hat{s}_b$) and imposing some reduction on the applied research subsidy rate. When the misclassification level exceeds the threshold and a tax on applied research is not possible, we cannot restore the optimal policy ($\hat{s}_a, \hat{s}_b$) but can maximize welfare by setting the applied subsidy to 0 and finding a second-best policy level for s_b.

The welfare level achieved for each level of misclassification is plotted in the right panel of Figure 13. Importantly for policymakers, the welfare gains arising from a type-dependent subsidy are robust to a substantial amount of misclassification before falling. Loosely speaking, the welfare gains associated with a type-dependent research subsidy are roughly the same for any level of misclassification below 50%.[30]

Figure 13: Optimal Policies & Welfare with Misclassification of Research



5.6 Academic Policy

An important question is whether and how the presence of a public research sector affects the optimal policy choices of the social planner. One possibility is that the public sector provides a more targeted

[30] Chen, Liu, Suárez Serrato, and Xu (2018) analyze the effects of a Chinese policy that awards substantial corporate tax cuts to firms that increase R&D investment over a given threshold. They find, amongst others, that about 30% in the resulting increase of R&D is due to relabeling of administrative expenses.

policy tool and is, therefore, a substitute for incentivizing private basic research efforts. Tables 11 and 12 explore these policy choices in several steps.

TABLE 11: OPTIMAL ACADEMIC POLICIES (IN PERCENTAGES)

Policy	ψ_a	ψ_b	R/Z	τ_a	τ_b^e	τ_u	L_p	L_b	L_u	L_e	L_a	α	g	β
Baseline	10.0	10.0	0.5	14.7	0.6	0.13	85.6	0.6	0.52	4.4	8.9	7.2	1.35	100.00
Soc Plan	-	-	-	12.0	3.8	1.89	83.1	4.5	2.22	3.4	6.8	35.4	1.88	104.61
Uniform	29.7	29.7	0.5	16.7	1.5	0.10	82.1	1.5	0.49	5.3	10.7	13.4	1.68	100.64
Targeted	11.1	48.6	0.5	12.7	4.4	0.11	83.5	5.1	0.50	3.5	7.3	29.9	1.71	102.81
Unif Acad	**29.8**	**29.8**	**0.68**	**16.7**	**1.46**	**0.27**	**81.9**	**1.47**	**0.67**	**5.3**	**10.7**	**15.4**	**1.70**	**100.70**
Unif Acad+	**24.3**	**24.3**	**3.24**	**15.5**	**0.89**	**2.80**	**81.7**	**0.86**	**3.19**	**4.7**	**9.6**	**24.4**	**1.88**	**102.88**

See Table 7 for variable definitions.

In the fifth row of Table 11, we allow the social planner to optimize not only on the uniform research subsidy to firms, but also on funding for the academic sector. The optimal funding for the academic sector calls for an increase in public investment by 0.18% of GDP, or a 40% increase with respect to the observed levels. Importantly, the optimal uniform subsidy to firms remains unchanged and suggests a potential complementary role for research policies targeted towards the public sector. Overall, the gains from moving to optimal academic policy are fairly modest and may reflect the delay in the applicability of academic research.

To explore this possibility, the sixth row of Table 11 relaxes the "Ivory Tower" assumption. In our model, academic research is assumed not to be immediately applicable to production. Its effects are borne out only through the effects of within-industry spillovers, with larger step sizes for applied innovations that build on it. Thus, there is a sense in which academic research is less immediately effective than private basic research, where innovations are directly used in production and spillovers are internalized when they fall within a firm's industrial scope. So we also include the results of an additional counterfactual exercise in which academic innovation is immediately applicable (Unif Acad+), putting it more on par with private basic research. Here we can see that the optimal level of academic funding goes up substantially, as do the associated welfare gains.[31]

Table 12 relaxes the assumption that the private and public research sectors have the same production functions. Unfortunately, we do not have data for our setting on the breakdown of costs of public and private research. However, the fact that there is a linkage between private and public research wages is a key insight that we exploit in this robustness check.[32] Consequently, we simulate the model with different levels of public research productivity, with cost parity as a lower bound on

[31]We also look at policies in terms of the optimal number of labs U. We find that the social planner would decrease the number of labs, from a baseline value of $U = 0.49$ to an optimal value of $U = 0.20$, yielding welfare gains of 0.23%. The intuition is that because of the high fixed cost the planner will find it optimal to consolidate labs, even at the expense of moving up the variable cost curve.

[32]The literature models the distinction between private and public research in terms of creative control (Lacetera, 2009; Howitt, 2000; Aghion and Tirole, 1997). The public sector, due to its non-profit nature, can essentially commit to leave scientists to pursue their own interests. Consequently if scientists value creative control they will have to be paid a wage premium in order to give it up. This feature has received empirical support in work by Stern (2004), who studies the job market for recent PhDs in biology. By using information on multiple job offers, he shows that wages are substantially lower in jobs that promise scientists a greater degree of freedom and disclosure of research.

the productivity of public researchers relative to private sector researchers. For each level of the productivity multiplier, we recompute the optimal uniform subsidy plus the academic spending-level policy, keeping other parameters the same as in the baseline estimation. Table 12 summarizes the results of this exercise.

Table 12: Optimal Subsidies With Various Academic Productivities

Academic Multiplier	Optimal Uniform Subsidy	Optimal Academic Spending	Welfare Gain
1x	29.8	0.68	0.7
2x	29.2	1.01	1.0
5x	27.6	1.54	1.9

Increasing the productivity of public research increases the optimal level of funding given to public research in the form of academic spending and substantially increases the associated welfare gains. The optimal uniform subsidy remains nearly constant, decreasing only slightly. There are two primary and countervailing forces at play. The first is that more academic basic research will increase the number of "hot" product lines, thus increasing the returns to applied research by private firms. The second is that more academic spending on researchers will increase the wage in general equilibrium, thus increasing the costs of applied research. Our estimates suggest, therefore, that the optimal uniform subsidy remains a robust complementary tool with respect to policies focusing on the public research sector.

5.7 Extensions and Robustness Checks

In this section, we summarize the robustness of our estimation results and policy analysis to a number of variations on our sample and model. We provide in Appendix G the implied parameter estimates, the match between the model and the data moments, the social planner's allocation, and the allocations that result from the different policy tools.

Heterogeneous industries and firm types We re-estimate and evaluate the policy implications of extensions of the model featuring (i) industry heterogeneity and (ii) persistent firm types. To address concerns about industry heterogeneity, we split the economy into two groups of industries that are differentiated by their parameters governing basic research costs and benefits. To address concerns about firm heterogeneity, we introduce a firm type, that is fully persistent, and that determines a firm's basic research cost structure. The implications for policy, reported in Tables 18 and 21 of Appendix G.1 and G.2, are robust across these extensions.

Research types and institutions We also re-estimate two versions of the model without the distinction between research efforts: (i) we eliminate private basic research but still maintain public basic research; (ii) we eliminate basic research altogether, both private and public, and obtain a model with a single type of research. The estimates and policy implications are reported in Tables 24 and 27 of Appendix G.3. It turns out that the existence of investment in basic research by the private and public sectors is important for our policy conclusions. Without these distinctions in research investment, and their associated spillovers, optimal policy would actually discourage research relative to the baseline economy.

Limited diffusion of private basic research We then consider two factors related to public basic research. First, research produced in the public sector is a public good, while private research may be kept secret by firms. Therefore, we consider a version of the model in which private basic research still benefits the firm but makes only a limited contribution to future economic growth. That is, firms keep a fraction of their research findings secret, and, hence, future innovators cannot build on that knowledge. The policy implications are depicted in Figure 15 of Appendix G.4.

Spillovers from applied research and CRRA parameter As a final set of robustness checks, we first consider whether applied research is necessarily targeted to specific product lines/industries. We extend the cross-industry spillover to applied research as well and provide a quantification of the model fit as a function of different applied step sizes in Figure 16 of Appendix G.5. Second, since the identification of the Constant Relative Risk Aversion (CRRA) parameter is indirect, we also perform in Table 31 of Appendix G.6, a robustness check in which we fix the parameter to a standard value of the literature and re-estimate the model.

6 Conclusion

In this paper, we distinguished between basic and applied research investment and identified the spillovers associated with each. We built a new micro-founded model of endogenous growth with two types of research investment (basic and applied) and two types of entities (private firms and public research labs). Our quantitative analysis highlighted the importance of these distinctions. First, in the competitive equilibrium, there is an overinvestment in applied research and underinvestment in basic research. As a result, imposing a uniform research subsidy does not generate the expected welfare improvement due to inefficient cross-subsidization of applied research. A key message of our paper is, therefore, that such policies can accentuate the dynamic misallocation in the economy. Second, there are important complementarities between public and private research investments. When public research labs produce more basic knowledge, private firms build on this and produce more impactful applied innovations.

Overall, our analysis relative to the uniform research subsidy resembles a reform of the French R&D policy. In 2008, the French government introduced a 30% tax credit for all of firms' R&D related expenditures. While the new system represented a significant subsidy to private R&D, with an annual budget of five billion Euros, its effectiveness in boosting innovation has been widely criticized (Larousserie, 2015).

Our findings can account for the limited impact of such policies and to the need for targeting basic research more directly. Our paper took a first step in trying to quantify the inefficiencies regarding different types of research and innovation efforts. There are still important open questions awaiting further study. Amongst them, our analysis highlighted the need for a better understanding of the public research sector, the design of the patent system, and the incorporation of firm heterogeneity and non-R&D firms. We hope that further structural work will be undertaken to enhance our understanding of the aforementioned issues, which can then guide the relevant innovation policies.

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Appendix

A Data Organization

Data Organization

We first identify the ownership status of each firm in the economy and the head of the group with which the firm is affiliated. Indeed, our data source (LIFI) defines a group as a set of firms controlled, directly or indirectly, by the same entity (the head of the group). We rely on a formal definition of control, requiring that a firm holds directly, or through cross-ownership, at least 50% of the voting rights in another firm's general assembly. We do not expect this to be a major source of bias in our sample as most French firms are private and ownership concentration is strong even among listed firms. Firms that do not conform to this definition are classified as stand-alone firms.

We then match the ownership information to our balance-sheet data and to our survey on lines of business within firms. We drop firms that appear in the ownership data but for which we cannot find balance-sheet information. We also delete as outliers firm-year observations whose ROA falls outside a multiple of five of the interquartile range and firms that report 0 employment or which have negative sales. Based on our two sources of information, we identify the main line of business from the balance sheets and the different segments of the firm from the survey on lines of business. For computational convenience we create a new firm-group identifier that allows us to aggregate at the same time business groups, business groups with multi-divisional firms, exclusively multi-divisional firms, and true stand-alone firms. We then define four measures of multi-market activity. The first measure counts each market in which the firm-group is present either via its ownership links or its multi-divisional structure. The second measure counts each market in which the firm-group is present with at least 9 employees via its ownership links or its multi-divisional structure. The third measure counts each market in which the firm-group is present exclusively via its ownership links. The final measure counts each market in which the firm-group is present exclusively via its ownership links and excluding financial activities.

We then define firm characteristics from balance-sheet data. There are three possible organizational types and comparison issues might arise. Taking the firm as the economic unit of interest has the advantage of simplicity since information is directly available in the balance sheets. However, this method has the disadvantage of not being comparable across organizational types. Indeed, most information for multi-divisional firms is aggregated across lines of segment, whereas firms belonging to business groups have market-specific information. Similar to existing studies by the Ministry of Research (Dhont-Peltrault and Pfister, 2011), we decided to aggregate the information to the economic unit at the highest level of control: the firm level for stand-alone and multi-divisional firms, and the business group level for firms affiliated through majority ownership.[33]

In a final step, we match the firm's balance-sheet and patent information to information contained in the R&D Survey. We focus on firms for which we have R&D information. Again we aggregate at the highest level of control. As before, one has to be cautious in aggregating on the basis of variables that might be prone to double-counting. When constructing information on the basic R&D intensity of a firm, this is not the case, as we are focusing exclusively on "internal" research expenditures. Therefore, if a member of the group contracts out research with another member of the group, then one will be counted as "external" research expenditures and the other one as "internal" expenditures. To correct for outliers in the dependent variable, we drop firm-year observations whose basic research intensity, conditional on positive basic research, falls outside a multiple of five of the interquartile range. In addition, we exclude firm-year observations whose total R&D to sales ratio falls outside a multiple of five of the interquartile range.[34]

[33] In addition to the economic rationale for constructing the data at the highest level of control there is also a legal argument. Indeed most public administrations and tribunals define the eligibility of firms for subsidy programs with respect to the business groups to which they belong.

[34] Alternatively, we exclude firm-year observations whose basic to applied R&D ratio falls above the 99^{th} percentile of the distribution. The results are qualitatively similar.

Policy Environment

It is useful to describe the policy environment in France during the period of our data. The share of GDP devoted R&D expenditures in France was on average 2.2% between 2000 and 2006. Innovation policy during the sample period featured a mix of measures to support R&D investment of firms through public financing. The main instrument to stimulate private innovation activity during that period consisted of approximately 2.5 billion euros of yearly subsidies allocated to firms either through ministries or government agencies such as OSEO-ANVAR. Note that our R&D survey allows us to directly measure this form of public financing in our sample. Finally, the R&D tax credit system was seen by the government as a secondary policy measure until a major reform in 2008 that increased the base and the rate of the the tax credit.

Variable List

All variables are organized and computed according to the method set out in the previous section. To summarize, we decided to aggregate the information to the economic unit at the highest level of control: the firm level for stand-alone and multi-divisional firms, and the business group level for firms affiliated through majority ownership. In the remainder of the document, we will, for the sake of notational convenience, refer generically to firms.

- *Basic Research Intensity:* total basic research by firm i in year t divided by total applied research of firm i in year t. The formulation of the survey questions related to the type of research undertaken is directly derived from the definitions provided by the Frascati Manual;

- *# of Industries:* sum of all distinct SIC codes within firm i in year t irrespective of organizational form (business group or multi-divisional structure). Industries are successively defined at the 4-,3-,2- and 1-digit SIC levels;

- *# of Industries - Weighted Sum:* weighted sum of all distinct bilateral 1-digit SIC links within firm i in year t. Weights are computed on the basis of the empirical frequency of each bilateral SIC link in each year t;

- *# of Patent Classes Applied:* sum of cumulated distinct patent-class applications within firm i in year t. Cumulated patent-class applications are computed for the period leading from 1993 to year t. Patent classes are successively defined at the 5,4,3,2 and 1-digit levels (EPO Classification);

- *# of Patent Classes Granted:* sum of cumulated distinct patent-class grants within firm i in year t. Cumulated patent-class grants are computed for the period leading from 1993 to year t. Patent classes are successively defined at the 5-,4-,3-,2- and 1-digit levels (EPO Classification);

- *Financial Int.:* binary indicator equal to 1 if firm i in year t is present in a financial industry, 0 otherwise;

- *Foreign HQ:* binary indicator equal to 1 if the headquarters of firm i in year t are located outside France, 0 otherwise;

- *Market Share:* weighted average of total sales of firm i, year t in industry k divided by total industry sales in year t. Weights are computed on the basis of the industry share of employment within firm i in year t;

- *Outsourcing to Univ.:* binary indicator equal to 1 if firm i in year t has outsourced R&D to French universities, 0 otherwise;

- *Profitability - ROA:* weighted average of EBIDTA divided by total fixed assets of all subsidiaries within firm i in year t. Weights are computed on the basis of the subsidiaries' share of employment within firm i in year t;

- *Profitability - ROS:* weighted average of EBIDTA divided by total sales of all subsidiaries within firm i in year t. Weights are computed on the basis of the subsidiaries' share of employment within firm i in year t;
- *Public R&D Funds:* binary indicator equal to 1 if firm i in year t has received French public funds, 0 otherwise;
- *Research Area:* weighted average of the share of respectively biotech / software / environmental research in research expenditures in firm i in year t. Weights are computed on the basis of the subsidiaries' share of total R&D within firm i in year t;
- *Total Employment:* total employment of firm i in year t;
- *IV - State Present in 1986:* binary indicator equal to 1 if the French state had a non-zero equity stake in firm i in 1986;
- *IV - SOE in 1986:* binary indicator equal to 1 if the French state had a controlling equity stake in firm i in 1986.

Descriptive Statistics

Table 13 provides the descriptive statistics of the key variables.

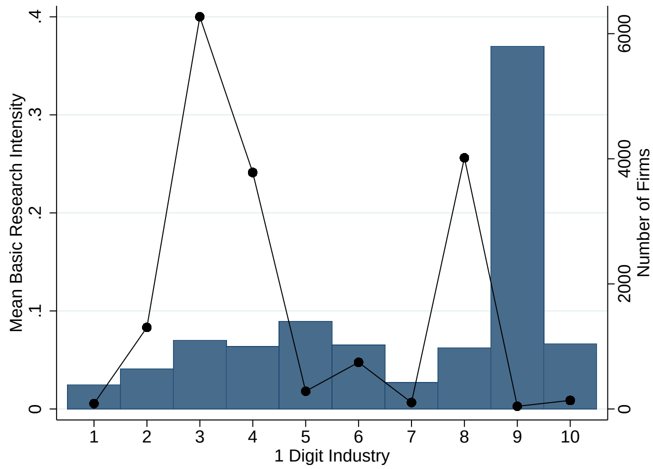
TABLE 13: DESCRIPTIVE STATISTICS

Variable	Mean	25th Percentile	Median	75th Percentile	Standard Deviation	Min	Max	N
R&D Investment								
R&D To Sales	0.11	0.01	0.04	0.14	0.17	0.00	0.86	13708
Basic Research Intensity	0.06	0.00	0.00	0.02	0.19	0.00	1.57	13708
Number of Industries								
1-Digit SIC	2.21	1	2	3	1.48	1	10	13708
4-Digit SIC	4.97	1	2	5	8.87	1	130	13708
Balance Sheet								
Total Employment	1497.88	24	93	506	8445.93	1	195746	13708
Return on Sales	0.032	0.02	0.07	0.13	0.63	-39.39	7.36	13708
Age	21.17	8.79	18.92	30.55	14.97	0	86	13708
Ownership Structure								
Financial Intermediary	0.05	0	0	0	0.22	0	1	13708
Foreign HQ	0.23	0	0	0	0.41	0	1	13708
Public and Private R&D								
Public Subsidy to Private Investment	0.09	0	0	0.04	0.4	0	30.9	13708
Collaboration with Universities	.15	0	0	0	0.36	0	1	13708

Note: Pooled data for the period 2000-2006. *R&D To Sales* is defined as the ratio of total firm research and development experndirures to total firm sales. *Basic Research Intensity* is defined as the ratio of total firm investment in basic research to total firm investment in applied research. *Number of Industries* is the sum of all distinct SIC codes within the firm. *Return on Sales* is the ratio of earnings before interest, taxes, depreciation and amortization to total firm sales. *Total Employment* total employment of the firm. *Age* is the difference between the current year and the year of the firm's incorporation. *Financial Intermediary* binary indicator equal to 1 if the firm is present in a financial industry, 0 otherwise. *Foreign HQ:* binary indicator equal to 1 if the headquarters of the firm are located outside France, 0 otherwise. *Public Subsidy to Private Investment* binary indicator equal to 1 if the firm has received French public funds for innovation expenditures, 0 otherwise. *Collaboration with Universities* binary indicator equal to 1 if the firm has received French public funds for innovation expenditures, 0 otherwise.

Figure 14 provides a description of industry patterns in basic research investment. The horizontal axis on the left measures average basic research intensity within an industry (bars) and is defined as the ratio of basic to applied research investment. The right-axis reports the number of firm-year observations within an industry and is plotted through the connected line.

Figure 14: Basic Research Intensity Across Industries



The figure plots the number of innovative firms (right axis, black line), and their average basic research intensity across industries (left axis, blue bars). Activity classification: 1 "Agriculture", 2 "Food and Textile Industries", 3 "Manufacture of chemicals, metals and machinery", 4 "Manufacture of electrical and transport equipment", 5 "Construction and Utilities", 6 "Wholesale and retail trade", 7 "Transport, Communication and Financial Activities", 8 "Professional, scientific and technical activities", 9 "Education and Human Health Activities", 10 "Arts, entertainment and others"'.

Across firms average basic research intensity is 6.5%, and the share of firms with positive basic research investment is 25%. At the same time, the Figure shows that there is heterogeneity in research investment across industries. Basic research intensity in most industries lies between 3% to 8-9%. It is highest for firms in manufacturing activities such as "Manufacture of chemicals, metals and machinery" and scientific activities ("Professional, scientific and technical activities"). The latter industry is dominated by information technology firms. These industries are also the largest contributors to basic research expenditures in aggregate terms. Basic research intensity is lowest for agriculture and finance activities. Finally, the figure suggests an outlier in terms of basic research investment is constituted by the industry of "Education and Human Health Activities." A closer inspection of the data reveals that most of these firms within that category are pharmaceutical/biotech companies. At the same time, their importance within the French economy seems to be rather limited in aggregate terms. The spike represents only 23 firms out of a total sample of 6,763 firms (representing less than 1% of aggregate basic research investment). In the empirical analysis we show that our estimates for the cross-industry spillover are robust to industry heterogeneity in basic research investment. First, our estimate is unaffected by the exclusion of the 23 firms in the biotech industry. Second, we control for other types of industry heterogeneity by including industry fixed effects as well as controls related to the field of research investment. Finally, we extend our model to allow for industries with differing basic research technologies.

B Robustness Check Related to Across Industry Spillover

Table 14 further addresses concerns about the measurement of the cross-industry spillover. We focus our discussion on the economic interpretation of the links between industries. One concern is that industries are not necessarily symmetric in their innovation incentives and their network links. Another concern is that the measurement of multi-industry presence could be complicated by artificial links either due to links with financial services or due to shell companies.

Table 14: Basic Research and Multi-Market Activity - Measurement

	(1)	(2)	(3)	(4)
Log # of Industries	0.032***	0.032***		
	(0.006)	(0.006)		
Log # of Industries (Size Condition)			0.026***	
			(0.007)	
Log # of Industries (Weighted Links)				0.007***
				(0.001)
Log Employment	0.003**		0.004***	0.003***
	(0.001)		(0.001)	(0.001)
Log Employment (Excluding Financials)		0.003**		
		(0.001)		
Financial Activities		-0.004		
		(0.009)		
HQ Financial		0.011		
		(0.017)		
HQ Foreign			-0.013***	
			(0.005)	
Employment Concentration Across Industries (HHI)			0.002	
			(0.015)	
Year & Organization FE	Yes	Yes	Yes	Yes
N	13708	13693	13708	13708

Notes: Pooled data for the period 2000-2006. Estimates are obtained using Tobit models and relate to the marginal effect of the variables at the sample mean. *Basic Research Intensity* is defined as the ratio of total firm investment in basic research divided by total firm investment in applied research. *Log # of Industries* is the number of distinct SIC codes in which a firm is present. *Log # of Industries (Size Condition)* excludes activities with less than 10 employees from the count of distinct SIC codes. *Log # of Industries (Weighted Links)* inversely weights the bilateral links of each firm by the frequency of the link in the population of French firms. *Log Employment* is the total employment of firms. *Log Employment (Excluding Financials)* excludes 2 digit SIC codes associated to financial activities from the total employment of firms. *Financial Activities* is a binary variable equal to 1 if the firm is active in 2 digit SIC codes associated to financial activities. *HQ Financial* is a binary variable equal to 1 if the head of the group is a financial holding company. *HQ Foreign* is a binary variable equal to 1 if the head of the group is a foreign holding company. *Employment Concentration Across Industries (HHI)* is the Herfindahl index of within firm employment across 1 digit SIC activities. *Share in Software/Biotech/Materials* is the share of R&D investment into the respective areas of research. *Industry fixed* denote 1 digit SIC activities. *Year FE* denotes year fixed effects, and *Organization FE* denotes whether the firm operates its activity as a conglomerate or as a business group. See the Appendix for the definition of variables. Robust standard errors clustered at the firm level are in parentheses. One star denotes significance at the 10% level, two stars denote significance at the 5% level, and three stars denote significance at the 1% level.

Column (2) addresses the concern that the link between research investment and multi-industry presence is driven by financial activities. To do so, we recompute multi-industry presence and size by excluding links to banks, insurance companies, and asset management companies. In addition, we explicitly control for the presence of financial activities within the group (either as a subsidiary or as the head of the group). The estimates are unaffected.[35] Column (3) tests whether links in our data are over-estimated due to artificial companies with no real economic activity. To do so, we recompute multi-industry presence and size by considering only activities with at least 10 employees. We also add two variables in the specification that should capture the possibility of spurious industry presence. The first is a dummy variable on whether the headquarters are foreign. The second is a Herfindahl index related to the concentration of employment within a given industry of the firm's portfolio. Again we find that our estimates are very similar in magnitude and precision. Column (4) tries to address concerns about the symmetric treatment of industry links in our analysis. This could be problematic if our multi-industry measure captures for instance integrated supply chains, rather than a larger breadth of a firm's activities. We go back to our data on the population of French firms and measure the frequency of each distinct activity pair. We then inversely weight the distinct bilateral links of our innovative firms. Intuitively, this measure will give less weight to a bilateral industry link if it is very frequent, and a higher weight if it is relatively rare. Computations and estimates are robust across all industry definitions but we only report the benchmark 1 digit case. The estimate suggests that this alternative measure of multi-industry presence is still positively correlated to basic research intensity. The difference in the point estimate is due to the difference in the support of the weighted industry variable, but the magnitude of a standard deviation increase on basic research incentives remains comparable (.019 increase in basic research intensity for the estimate using the weighted measure as opposed to .022 for the estimate using the count measure).

[35] Note that this result is not surprising given that the positive correlation also existed for more granular decompositions at the 3 and 4 digit SIC levels.

C Robustness Check Related to Patent Citation Patterns

Table 15 provides additional robustness checks for the estimates on the cool-down rate of patents originating from basic and applied research. We not only relax the restriction of same industry of use, but also change the time horizon in which we measure patent quality.

The top panel of the table measures quality of follow-up patents by computing the 5-years-forward citations of the citing patents and is measured for patents granted in the period 1975-1985. The bottom panel re-classifies university patents that were defined as private depositors. In both cases, results are unchanged, with a citation difference between public and private patents that becomes statistically non-significant at year 8. Indeed, in France, most of the academic patents are accounted for in the "public" category. French universities generally manage their patents through public research institutions with which academics are typically affiliated, one example being the CNRS.

Table 15: 2nd Generation Citation Patterns for Public and Private Patents

Age	1	2	3	4	5	6	7	8	9	10
5-Yr-Forward Citations	.15**	.16**	.28***	.16**	.22**	.15**	.33***	.08	.18	.15
	(0.07)	(0.07)	(0.08)	(0.06)	(0.07)	(0.07)	(0.11)	(0.08)	(0.11)	(0.12)
10-Yr-Forward Citations	.3**	.3**	.62***	.28**	.42**	.23	.71***	.08	.39	.15
Including Univ.	(0.15)	(0.15)	(0.17)	(0.14)	(0.18)	(0.17)	(0.25)	(0.16)	(0.25)	(0.24)

*Note:*The table computes *Average Citations of Citing Patents* computing the 5-years-forward citations of the citing patents and re-classifying university patents as public patents. The table reports differences in citation patterns using two sample t-tests with unequal variances. One star denotes significance at the 10% level, two stars denote significance at the 5% level, and three stars denote significance at the 1% level

D Theoretical Proofs

Spillovers Recall that the distribution of the number of spillovers is given by the geometric distribution

$$F^n = (1-s)s^n \quad \text{where} \quad s = \frac{p}{1+p}$$

Thus by the properties of the geometric distribution, the expected number of spillovers overall is

$$\mathbb{E}[n] = \sum_{n\geq 0} n \cdot F^n = \frac{s}{1-s} = p$$

For a firm operating in m (out of M) industries that gets n spillovers, the probability of getting k utilized spillovers is

$$B_m^{k|n} = \binom{n}{k}\left(\frac{m}{M}\right)^k\left(\frac{M-m}{M}\right)^{n-k} = \binom{n}{k}\left(\frac{M-m}{M}\right)^n\left(\frac{m}{M-m}\right)^k$$

It is useful to know the following binomial sum

$$\sum_{n\geq k}\binom{n}{k}y^n = \frac{y^k}{(1-y)^{k+1}}$$

Combining these, we find the the utilized spillover (k) distribution for a firm operating in m industries

$$\begin{aligned}
F_m^k &= \sum_{n\geq k} B_m^{k|n}\cdot F^n = \sum_{n\geq k}\binom{n}{k}\left(\frac{M-m}{M}\right)^n\left(\frac{m}{M-m}\right)^k(1-s)s^n \\
&= (1-s)\left(\frac{m}{M-m}\right)^k\sum_{n\geq k}\binom{n}{k}\left[s\left(\frac{M-m}{M}\right)\right]^n \\
&= (1-s)s^k\left(\frac{m}{M}\right)^k\left(\frac{M}{sm+(1-s)M}\right)^{k+1} \\
&= \left(\frac{(1-s)M}{sm+(1-s)M}\right)\left(\frac{sm}{sm+(1-s)M}\right)^k
\end{aligned}$$

Then we can summarize these results with

$$F_m^k = (1-s_m)s_m^k \quad \text{where} \quad s_m = \frac{sm}{sm+(1-s)M}$$

The mean of this distribution is

$$\mathbb{E}_m[k] = \frac{s_m}{1-s_m} = \frac{s}{1-s}\frac{m}{M} = p\frac{m}{M}$$

Finally, in terms of p only, we get

$$F_m^k = \left(\frac{M}{pm+M}\right)\left(\frac{pm}{pm+M}\right)^k \quad \text{where} \quad s_m = \frac{pm}{pm+M}$$

Production As the downstream production technology is unchanged in the generalized model and we continue to impose symmetry across the industries. This implies that

$$P_i = P = \frac{1}{M} \qquad \text{and} \qquad Y_i = Y = Z. \tag{29}$$

Henceforth, we can drop the industry index i. The perfectly competitive firm that produces midstream good Y_i takes equilibrium prices P and p_j as given while maximizing its profit

$$\max_{y_j} \left\{ P \left[\int_0^1 y_j^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 p_j y_j dj \right\}.$$

This maximization leads to the following inverse demand for upstream good j

$$p_j = P \left(\frac{Y}{y_j} \right)^{\frac{1}{\varepsilon}}.$$

Monopolist in product line j, j has productivity q_j. The firm takes the demand function for its product as given and solves the following maximization problem

$$\pi_j = \max_{y_j} \left\{ P Y^{\frac{1}{\varepsilon}} y_j^{\frac{\varepsilon-1}{\varepsilon}} - \frac{w}{q_j} y_j \right\}$$

This delivers the following optimal quantity

$$y_j = \left[\frac{1}{M} \left(\frac{\varepsilon - 1}{\varepsilon} \right) \left(\frac{q_j}{w} \right) \right]^{\varepsilon} Z$$

Plugging this into the production function for midstream goods, we find a relationship between wage w and aggregated productivity $\bar{q} \equiv \left(\int q_j^{\varepsilon-1} dj \right)^{\frac{1}{\varepsilon-1}}$

$$w = \frac{1}{M} \left(\frac{\varepsilon - 1}{\varepsilon} \right) \bar{q} \tag{30}$$

With this, we can greatly simplify the expression of the firm's quantity and price choices as a function of its normalized productivity $\hat{q}_j = q_j / \bar{q}$

$$y_j = \hat{q}_j^{\varepsilon} Z \qquad \text{and} \qquad p_j = \frac{1}{M \hat{q}_j}$$

Denote variables normalized by Z/M with a "$\sim$". Then the normalized profit and labor are given by

$$\tilde{\pi}_j = \frac{\hat{q}_j^{\varepsilon-1}}{\varepsilon} \qquad \text{and} \qquad l_j = \frac{\hat{q}_j^{\varepsilon-1}}{\tilde{w}} \left(\frac{\varepsilon - 1}{\varepsilon} \right). \tag{31}$$

where $\tilde{w}$ is the normalized wage. Note that by construction $\int \hat{q}_j^{\varepsilon-1} dj = 1$. As a result, we integrate 31 over j to find profit share and production labor share as

$$\frac{M \int_0^1 \pi_j dj}{Z} = \frac{1}{\varepsilon} \qquad \text{and} \qquad \frac{w L_P}{Z} = \frac{\varepsilon - 1}{\varepsilon}. \tag{32}$$

Finally, we combine 30 and 32 to find the final output as a function of aggregate productivity $\bar{q}$ and total production labor L_P :

$$Z = \bar{q} L_P / M.$$

Proof of Lemma 1 Let $\mathcal{F}_H(\cdot,t)$ and $\mathcal{F}_C(\cdot,t)$ be the aggregate product cumulative measures by type (hot or hold) at time t. For a small time step Δ, hot distribution $\mathcal{F}_H(\cdot,t)$ will satisfy

$$\begin{aligned}\mathcal{F}_H(\hat{q},t+\Delta) =&\mathcal{F}_H(\hat{q}/(1+\Delta g),t) - \Delta\tau\left[\mathcal{F}_H(\hat{q}/(1+\Delta g),t) - \mathcal{F}_H(\hat{q}/(1+\Delta g)-\eta,t)\right]\\ &+\Delta\tau_b^e\mathcal{F}_C(\hat{q}/(1+\Delta g)-\eta,t) - \Delta\zeta\mathcal{F}_H(\hat{q}/(1+\Delta g),t) + \Delta\tau_b^d\mathcal{F}_C(\hat{q}/(1+\Delta g),t)\end{aligned}$$

Similarly, the cold distribution $\mathcal{F}_C(\cdot,t)$ will satisfy

$$\begin{aligned}\mathcal{F}_C(\hat{q},t+\Delta) =&\mathcal{F}_C(\hat{q}/(1+\Delta g),t) - \Delta\tau_a\left[\mathcal{F}_C(\hat{q}/(1+\Delta g),t) - \mathcal{F}_C(\hat{q}/(1+\Delta g)-\lambda,t)\right]\\ &-\Delta\tau_b\mathcal{F}_C(\hat{q}/(1+\Delta g),t) + \Delta\zeta\mathcal{F}_H(\hat{q}/(1+\Delta g),t)\end{aligned}$$

Finally, for $i\in\{H,C\}$, calculating

$$\dot{\mathcal{F}}_i(\hat{q}) = \frac{\mathcal{F}_i(\hat{q},t+\Delta)-\mathcal{F}_i(\hat{q},t)}{\Delta}$$

and taking the limit as $\Delta\to 0$ yields the desired flow equations. Note that for this we use

$$\frac{\mathcal{F}_i(\hat{q}/(1+\Delta g),t)-\mathcal{F}_i(\hat{q},t)}{\Delta} = -g\hat{q}[\partial\mathcal{F}_i(\hat{q})/\partial\hat{q}]$$

Proof of Proposition 2. Let $\mathcal{F}(\cdot,t)$ be the distribution over q at time t. Similarly, let $\mathcal{F}_H(\cdot,t)$ and $\mathcal{F}_C(\cdot,t)$ be the product type (hot/cold) conditional distributions. Thus, we will have $\mathcal{F}(q,t) = \alpha\mathcal{F}_H(q,t)+(1-\alpha)\mathcal{F}_C(q,t)$. The evolution of the aggregated productivity index $\bar{q}$ is then given by

$$\begin{aligned}\bar{q}^{\varepsilon-1}(t+\Delta t) =& \int_0^\infty q^{\varepsilon-1}d\mathcal{F}(q,t+\Delta t)\\ =&\alpha\int_0^\infty q^{\varepsilon-1}d\mathcal{F}_H(q,t+\Delta t) + (1-\alpha)\int_0^\infty q^{\varepsilon-1}d\mathcal{F}_C(q,t+\Delta t)\\ =&\alpha\int_0^\infty\left[\Delta\tau\,(q+\eta\bar{q})^{\varepsilon-1}+(1-\Delta\tau)q^{\varepsilon-1}\right]d\mathcal{F}_H(q,t)\\ &+(1-\alpha)\int_0^\infty\left[\Delta\tau_a\,(q+\lambda\bar{q})^{\varepsilon-1}+\Delta\tau_b^e\,(q+\eta\bar{q})^{\varepsilon-1}+(1-\Delta\tau)q^{\varepsilon-1}\right]d\mathcal{F}_C(q,t)\end{aligned}$$

Thus the differential is

$$\begin{aligned}\frac{\bar{q}^{\varepsilon-1}(t+\Delta t)-\bar{q}^{\varepsilon-1}(t)}{\Delta} =&\alpha\int_0^\infty\tau\left[(q+\eta\bar{q})^{\varepsilon-1}-q^{\varepsilon-1}\right]d\mathcal{F}_H(q,t)\\ &+(1-\alpha)\int_0^\infty\left(\tau_a\left[(q+\lambda)^{\varepsilon-1}-q^{\varepsilon-1}\right]+\tau_b^e\left[(q+\eta)^{\varepsilon-1}-q^{\varepsilon-1}\right]\right)d\mathcal{F}_C(q,t)\end{aligned}$$

and the normalized differential is

$$\begin{aligned}\frac{\bar{q}^{\varepsilon-1}(t+\Delta t)-\bar{q}^{\varepsilon-1}(t)}{\Delta\bar{q}^{\varepsilon-1}(t)} =&\alpha\int_0^\infty\tau\left[(\hat{q}+\eta)^{\varepsilon-1}-\hat{q}^{\varepsilon-1}\right]d\mathcal{F}_H(\hat{q},t)\\ &+(1-\alpha)\int_0^\infty\left(\tau_a\left[(\hat{q}+\lambda)^{\varepsilon-1}-\hat{q}^{\varepsilon-1}\right]+\tau_b^e\left[(\hat{q}+\eta)^{\varepsilon-1}-\hat{q}^{\varepsilon-1}\right]\right)d\mathcal{F}_C(\hat{q},t)\end{aligned}$$

Finally, the growth can be expressed compactly as

$$g = \frac{\alpha\tau\mathbb{E}_{\hat{q}}^H\left[(\hat{q}+\eta)^{\varepsilon-1}-\hat{q}^{\varepsilon-1}\right]+(1-\alpha)\left(\tau_a\mathbb{E}_{\hat{q}}^C\left[(\hat{q}+\lambda)^{\varepsilon-1}-\hat{q}^{\varepsilon-1}\right]+\tau_b^e\mathbb{E}_{\hat{q}}^C\left[(\hat{q}+\eta)^{\varepsilon-1}-\hat{q}^{\varepsilon-1}\right]\right)}{\varepsilon-1}$$

This can also be rearranged into

$$g = \frac{\tau_a \left(\alpha \mathbb{E}^H_{\hat{q}} \left[(\hat{q}+\eta)^{\varepsilon-1} - \hat{q}^{\varepsilon-1} \right] + (1-\alpha) \mathbb{E}^C_{\hat{q}} \left[(\hat{q}+\lambda)^{\varepsilon-1} - \hat{q}^{\varepsilon-1} \right] \right) + \tau_b^e \mathbb{E}_{\hat{q}} \left[(\hat{q}+\eta)^{\varepsilon-1} - \hat{q}^{\varepsilon-1} \right]}{\varepsilon - 1}$$

■

Proof of Proposition 1.

Generally, the value of a firm with a product line portfolio $\hat{\mathbf{q}}$ operating in m industries can be expressed as

$$\begin{aligned} & r\mathcal{V}_t(\hat{\mathbf{q}}, m) - \dot{\mathcal{V}}_t(\hat{\mathbf{q}}, m) \\ = & \max_{a,b} \left\{ \begin{array}{c} \sum_{\hat{q}\in\hat{\mathbf{q}}} \frac{1}{\varepsilon} \hat{q}^{\varepsilon-1} \frac{Z_t}{M} - n w_t \left[(1-\psi_a) h_a(a) + (1-\psi_b)[h_b(b) + \mathbf{1}_{(b>0)}\phi] \right] \\ +na \left[\alpha \mathbb{E}^H_{\hat{q}} \mathcal{V}_t \left(\hat{\mathbf{q}} \cup \{\hat{q}+\eta\}, m \right) + (1-\alpha) \mathbb{E}^C_{\hat{q}} \mathcal{V}_t \left(\hat{\mathbf{q}} \cup \{\hat{q}+\lambda\}, m \right) - \mathcal{V}_t(\hat{\mathbf{q}}, m) \right] \\ +nb \sum_{k\geq 0} F^k_m \left[\mathcal{V}_t \left(\hat{\mathbf{q}} \cup \{\hat{q}+\eta\}^{1+k}, m \right) - \mathcal{V}_t(\hat{\mathbf{q}}, m) \right] \\ + \sum_{\hat{q}\in\hat{\mathbf{q}}} \tau \left[\sum_{\hat{q}\in\hat{\mathbf{q}}} \left[\mathcal{V}_t \left(\hat{\mathbf{q}} \setminus \{\hat{q}\}, m \right) - \mathcal{V}_t(\hat{\mathbf{q}}, m) \right] \right] \\ +x\frac{m}{M} \left[\alpha \mathbb{E}^H_{\hat{q}} \mathcal{V}_t \left(\hat{\mathbf{q}} \cup \{\hat{q}+\eta\}, m \right) + (1-\alpha) \mathbb{E}^C_{\hat{q}} \mathcal{V}_t \left(\hat{\mathbf{q}} \cup \{\hat{q}+\lambda\}, m \right) - \mathcal{P}'_m - \mathcal{V}_t(\hat{\mathbf{q}}, m) \right] \\ +x\left(1-\frac{m}{M}\right) \left[\alpha \mathbb{E}^H_{\hat{q}} \mathcal{V}_t \left(\hat{\mathbf{q}} \cup \{\hat{q}+\eta\}, m+1 \right) + (1-\alpha) \mathbb{E}^C_{\hat{q}} \mathcal{V}_t \left(\hat{\mathbf{q}} \cup \{\hat{q}+\lambda\}, m+1 \right) - \mathcal{P}_m - \mathcal{V}_t(\hat{\mathbf{q}}, m) \right] \\ +n\kappa \left[\mathbb{E}_{\hat{q}} \mathcal{V}_t \left(\hat{\mathbf{q}} \cup \{\hat{q}\}, m \right) - \mathcal{V}_t(\hat{\mathbf{q}}, m) \right] \\ +\kappa \left[-\mathcal{V}_t(\hat{\mathbf{q}}, m) \right] \end{array} \right\}. \end{aligned}$$

Intuitively, there is discounting at the rate r. The first line simply subtracts the instantaneous research expenditures from operating profits to obtain the net instantaneous profits. The second line expresses the change in firm value due to applied innovation. Notice that with applied innovation, we must form an expectation about how big the innovation size is going to be using the share of undepreciated product lines α. The third line expresses the change in firm value due to basic innovation in the initial industry and in the spillover industries. Note that spillovers will land in a random number of industries, but in expectation, the firm will be able to use innovations in $p \times \frac{m}{M}$ industries. The fourth line describes the change in firm value due to creative destruction, which happens at the rate τ. The fifth and sixth lines describe the effects of potentially buying out entrants in cases when the product line does or does not fall within one of the firm's active industries. In either case, the buyout price ($\mathcal{P}_m$ or $\mathcal{P}'_m$) is simply the present discounted production value to the incumbent firm, reflecting a case in which they have very low bargaining power. The last two lines describe the effects of exogenous destruction of product lines and the random redistribution of these to other firms.

Now, conjecture $\mathcal{V}_t(\hat{\mathbf{q}}) = \frac{Z_t}{M} \left[\sum_{\hat{q}\in\hat{\mathbf{q}}} V(\hat{q}_t) + nV_m \right]$. When we substitute the conjecture into the the above expression and using the prices

$$\begin{aligned} \mathcal{P}_m &= V_{m+1} + \mathbb{E}_{\hat{q},s} V\left(\hat{q}_{t+\Delta t} + \hat{s} \right) \\ \mathcal{P}'_m &= V_m + \mathbb{E}_{\hat{q},s} V\left(\hat{q}_{t+\Delta t} + \hat{s} \right) \end{aligned}$$

we find

$$(r-g)\, V_m = \max_{a,b} \left\{ \begin{array}{c} -\tilde{w} \left[(1-\psi_a) h_a(a) + (1-\psi_b)[h_b(b) + \mathbf{1}_{(b>0)}] \right] \\ +a \left[\alpha V^H + (1-\alpha) V^C + V_m \right] \\ +b\left(1 + p\frac{m}{M}\right) \left[V^H + V_m \right] \\ +x\left(1 - \frac{m}{M}\right) \left[V_{m+1} - V_m \right] \\ -\tau V_m + \kappa \mathbb{E}_{\hat{q}} V(\hat{q}_t) \end{array} \right\}.$$

and

$$V'(\hat{q}_t) g \hat{q} + \left[\tau + \kappa + r - g \right] V(\hat{q}_t) = \frac{1}{\varepsilon} \hat{q}^{\varepsilon-1}.$$

Note that the last expression is a differential equation as a function of $\hat{q}$. Then

$$V(\hat{q}_t) = \frac{\hat{q}_t^{\varepsilon-1}}{\varepsilon\left[r+\tau+\kappa+g\left(\varepsilon-2\right)\right]}.$$

This completes the proof. ■

Derivation of Multi-industry Distribution $\Gamma_{m,n}$.
We assume that when a firm loses its last product in a particular industry, it maintains a foothold there, in the sense that it still receives buy-out offers and can still directly use basic research relevant to that industry. When a firm loses all of its products or receives a destructive shock, it ceases to exist. We wish to find the joint distribution over the number of industries a firm is in and how many product lines it owns. For notational convenience, let us denote the basic research flow from m-industry firms by $\hat{b}_m = \mathcal{B}(\phi_m)b_m$. Let us also denote the expansion rate of a firm into a new industry by e_m. Here the expansion rate comes purely from buy-out offers by entrants. So given a per firm buy-out offer rate of x, a firm in m industries will expand at rate

$$e_m = x\left(\frac{M-m}{M}\right) = \left(\frac{\varsigma E a_e}{F}\right)\left(\frac{M-m}{M}\right)$$

Additionally, let the unconditional expected number of spillovers be denoted by $\rho_m = p\cdot\frac{m}{M}$. Then the flow equation for firms in m industries with n products is

$$\begin{array}{rcl}
\text{Outflow} & & \text{Inflow} \\
\left[\begin{array}{c} a_1+\hat{b}_1+\tau+\kappa \\ +e_1+\kappa \end{array}\right]\Gamma_{1,1} & = & a_e + 2\tau\Gamma_{1,2} \\
\left[\begin{array}{c} a_m+\hat{b}_m+\tau+\kappa \\ +e_m+\kappa \end{array}\right]\Gamma_{m,1} & = & 2\tau\Gamma_{m,2} + e_{m-1}\Gamma_{m-1,1} \text{ for } m\geq 2 \\
\left[\begin{array}{c} n\left(a_m+\hat{b}_m+\tau+\kappa\right) \\ +e_m+\kappa \end{array}\right]\Gamma_{m,2} & = & \left\{\begin{array}{c} \left(a_m+\hat{b}_m\left(1-\rho_m\right)+\kappa\right)\Gamma_{m,n-1} \\ +3\tau\Gamma_{m,n+1}+e_{m-1}\Gamma_{m-1,n} \end{array}\right\} \text{ for } m\geq 1 \\
\left[\begin{array}{c} n\left(a_m+\hat{b}_m+\tau+\kappa\right) \\ +e_m+\kappa \end{array}\right]\Gamma_{m,n} & = & \left\{\begin{array}{c} (n-1)\left(a_m+\hat{b}_m\left(1-\rho_m\right)+\kappa\right)\Gamma_{m,n-1} \\ +(n-2)\rho_m\hat{b}_m\Gamma_{m,n-2} \\ +(n+1)\tau\Gamma_{m,n+1}+e_{m-1}\Gamma_{m-1,n} \end{array}\right\} \text{ for } n\geq 3,\ m\geq 1
\end{array}$$

where we use the convention $\Gamma_{m,-1} = \Gamma_{m,0} = 0$ and $e_0 = 0$. The first line equates the outflows from $(m=1, n=1)$ that happen once the firm loses its product at the rate $\tau+\kappa$, acquires a new product line at the rate κ, innovates a new good at the rate $a_1+\hat{b}_1$ on average or expands into a new industry at the rate e_1. On the other hand, inflow happens from outsiders at the rate a_e and from the firms with 2 products that lose one of their products at the rate 2τ. Similar reasoning applies to the subsequent lines.

Using values for the $\Gamma_{m,n}$ distribution gives us the mass of firms in a given (m,n) state. The total mass of firms is then $F = \sum_{m=1}^{M}\sum_{n=1}^{\infty}\Gamma_{m,n}$. We ultimately want the mass of products in given industry state m. To get this we simply evaluate

$$\mu_m = \sum_{n=1}^{\infty} n\cdot\Gamma_{m,n}$$

Note that this derivation is for the case in which a single spillover arrives with probability p. It is straightforward to generalize this to our baseline setting in which a random number of spillovers is realized. ■

E Computer Algorithm Outline

Here we present the solution algorithm for a slightly simplified version of the baseline model. An equilibrium of this model is described by a system of five equations in the five variables $(\tau_a, \tau_b^e, \tau_b^d, \tilde{w}, g)$. This system can be evaluated using the following procedure:

1. Calculate α and the distribution of $\hat{q}$ using τ_a, τ_b^e, τ_b^d, and g according to equations (1) and (16).
2. Calculate g using, τ_a, τ_b^e, α, and the distribution over $\hat{q}$ with equation (21).
3. Calculate $V^H = \mathbb{E}^H_{\hat{q},\eta} V(\hat{q}+\eta)$ and $V^C = \mathbb{E}^C_{\hat{q},\lambda} V(\hat{q}+\lambda)$ using the relevant step size distribution and the type-specific productivity distributions.
4. Solve for the industry-specific value components V_m using the expression in equation (14). Using these, find a_m and b_m using the relevant first-order conditions and $\tilde{w}$.
5. Impose an upper bound on n and find the steady state product distribution $\Gamma_{m,n}$ using the flow equations in Appendix D.
6. Compute the updated values of τ_a, τ_b^e, and τ_b^d using equations (17) and (15).
7. The difference between the conjectured and updated values of τ_a, τ_b^e, τ_b^d, and g in conjunction with the labor market clearing differential from equation (19) constitute the five desired equations.

We use the trust-region dogleg method (the default in MATLAB) to solve this set of equations for a given set of parameters. To minimize the SMM objective function, we perform a search over the parameter space using a combination of a naive simulated annealing algorithm and a Nelder-Mead simplex (Nelder and Mead, 1965) algorithm. See Zangwill and Garcia (1981) for more information on solving systems of nonlinear equations.

F Quantitative Tables

F.1 Remaining Moments and Parameters

Profitability, Λ(19): Firm profitability is defined as the ratio of profits to sales. For a given panel of firms, this moment is given by

$$\Lambda(19) = \frac{1}{\varepsilon} - \mathbb{E}_{m,n,\hat{\mathbf{q}}}\left[\frac{\tilde{w}\left[h_a(a_m) + P_m(h_b(b_m) + \bar{\phi}_m)\right]}{\frac{1}{n}\sum_i \hat{q}_i^{\varepsilon-1}}\right].$$

Notice that there is one fixed component from the static production side that yields information on the value of ε and another from dynamic R&D expenditures that yields information on R&D cost and step-size parameters. Consistent with this, though this moment value arises from a multitude of factors, the major determinants are ε and the various fixed and variable R&D cost parameters.

Exit Rate, Λ(20): As exit occurs when firms either receive the exogenous destruction shock or lose their last product, the predicted exit rate will be

$$\Lambda(20) = \kappa + \tau \cdot \sum_m \Gamma_{m,1}.$$

However, for consistency, we simply use the value from the simulated firm sample. This moment serves primarily to determine the value of the rate of exogenous destruction κ. Additionally, as the bulk of equilibrium creative destruction τ arises from applied research, this also provides information on the applied R&D cost parameters ν_a and ξ_a.

Total Research Intensity, Λ(21): We have one moment to track the level of R&D overall: the ratio of total research labor expenditures spending to total production labor expenditures. Since research spending is proportional to n, R&D expenditures per product will be the same across firms with the same m, while employment will be a function of the portfolio of product qualities. Because the wage is common to both types of labor, this will simply be the ratio of R&D employment to production employment given by

$$\Lambda(21) = \mathbb{E}_{m,n,\hat{\mathbf{q}}}\left[\frac{\tilde{w}\left[h_a(a_m) + P_m(h_b(b_m) + \bar{\phi}_m)\right]}{\left(\frac{\varepsilon-1}{\varepsilon}\right)\frac{1}{n}\sum_i \hat{q}_i^{\varepsilon-1}}\right].$$

This moment gives us primarily information on the cost side of R&D–namely, the research production function parameters. It will also be influenced by the benefit side, in the form of innovation step sizes for basic and applied research, η and λ (see Table 16). However, these step-size parameters are largely determined by the citation evidence.

Firm Growth, Λ(22): We have a moment for the employment growth of firms. This is calculated conditional on the firm not exiting, as we do not observe the last period's growth rate for exiting firms. The moment is calculated by looking at the one-year growth rate of a firm's total employment. This moment is sensitive to the overall rates of exit and creative destruction and, as such, yields information on a broad array of parameters.

As seen in the Jacobian matrix in Table 16, one parameter for which firm growth shows particular distinction is the CRRA parameter γ. Loosely speaking, one function of this parameter is to govern the differential between the interest rate r and the growth rate g, which forms the basis of a firm's effective discount rate. As this identification is somewhat indirect, in the sense that it does not deal with consumer choice directly, we also perform a robustness check in which we fix the CRRA parameter to the standard value of $\gamma = 2$ and re-estimate the model.

Aggregate Growth, Λ(23): The aggregate growth rate gives information on the effectiveness of research spending, absent effects coming from the distribution of firm size and its relation to firm growth, particularly on innovation step sizes.

In our model, the household's welfare depends crucially on the level of aggregate growth, and hitting that moment is of particular importance. For that purpose, we boost the weighting on the aggregate growth moment.[36]

Firm Age, Λ(25-26): Firm age is highly correlated with firm size. We track the average age of firms for those above and below the median firm size. This yields information on entry and exit patterns, as well as on the rate of creative destruction. Moment $\Lambda(25)$ is the average age of firms below the median firm size, while moment $\Lambda(26)$ is the average age for those firms above it.

F.2 Jacobian of Estimation Moments

Table 16: Jacobian of Estimation Moments With Respect to Model Parameters

Parameter:	δ	γ	ε	p	η	λ	E	U	ν_a	ν_b	ξ_a	ξ_b	κ	$\bar{\phi}$	σ	ζ	ς
Moment:																	
Basic extensive $m = 1$	-1	-1	-6	0	13	-9	-4	1	-12	63	32	-26	-0	87	4	1	-0
Basic extensive $m = 2$	-1	-1	-5	1	13	-8	-4	1	-11	57	30	-25	-0	79	3	1	-0
Basic extensive $m = 3$	-1	-1	-5	1	12	-8	-4	1	-11	52	28	-23	-0	72	2	1	-0
Basic extensive $m = 4$	-1	-1	-5	2	11	-7	-3	1	-10	48	26	-22	-0	66	2	1	-0
Basic extensive $m = 5$	-1	-1	-5	2	10	-7	-3	0	-9	44	24	-21	-0	60	1	1	-0
Basic extensive $m = 6$	-1	-1	-4	2	10	-6	-3	0	-9	40	22	-20	-0	55	1	1	-0
Basic extensive $m = 7$	-1	-1	-4	3	9	-6	-3	0	-8	37	21	-19	-0	50	0	0	-0
Basic extensive $m \geq 8$	-1	-1	-4	3	8	-6	-3	0	-8	33	18	-18	0	44	-0	0	-0
Basic intensive $m = 1$	-1	-1	-2	0	18	-11	-3	1	-15	86	45	-32	-0	80	3	1	-0
Basic intensive $m = 2$	-1	-1	-2	1	17	-10	-3	1	-14	80	42	-30	-0	73	3	1	-0
Basic intensive $m = 3$	-1	-1	-1	2	16	-10	-3	1	-13	74	40	-29	-0	66	2	1	-0
Basic intensive $m = 4$	-1	-1	-1	2	15	-9	-3	1	-13	68	38	-28	-0	59	1	1	-0
Basic intensive $m = 5$	-1	-1	-1	3	14	-9	-2	1	-12	63	35	-27	-0	53	1	1	-0
Basic intensive $m = 6$	-1	-1	-0	3	14	-9	-2	1	-12	58	34	-26	-0	48	0	1	-0
Basic intensive $m = 7$	-1	-1	-0	4	13	-8	-2	1	-11	54	32	-25	-0	43	-0	1	-0
Basic intensive $m \geq 8$	-1	-1	-0	4	12	-8	-2	0	-11	49	29	-24	0	37	-1	1	-0
Mean m	-0	-0	0	0	0	-0	-1	-0	-0	0	0	-0	-0	0	0	-0	4
Mean m^2	-0	-0	0	0	0	-0	-1	-0	-1	1	1	-1	-0	1	0	-0	9
Return on sales	4	0	-23	-0	-14	3	7	-3	8	-16	-4	13	1	-18	-0	-3	-0
Exit Rate	-1	-1	-4	-0	-0	-0	1	0	3	-0	-4	0	1	-0	-0	0	-2
R&D/labor	3	-3	0	-2	5	8	5	5	1	4	4	1	4	4	-5	7	7
Employment growth	-1	-1	-3	0	2	4	0	-0	2	4	-2	-2	-0	4	0	-0	0
Aggregate growth	0	0	0	-0	-0	0	0	-0	0	-2	-1	1	0	-2	-0	-5	-0
Spillover differential	1	2	10	2	7	3	0	2	-1	10	6	-2	3	10	1	2	2
Age, small firms	-0	1	3	-1	-1	-1	-3	-0	-3	-0	2	-2	-1	0	-1	-0	2
Age, large firms	0	-0	4	1	1	1	-3	0	-3	2	6	-0	0	2	1	0	3
Public Citations Mean	1	1	2	-2	-9	10	5	-1	17	-44	-29	42	0	-41	-2	-1	-2
Public Citations RMS	1	1	2	-2	-8	10	5	-1	17	-43	-28	41	0	-40	-2	-1	-2
Private Citations Mean	1	1	2	-2	-12	14	5	-1	17	-42	-28	41	0	-39	-2	-1	-2
Private Citations RMS	1	1	2	-2	-11	14	5	-1	16	-40	-27	39	0	-37	-2	-1	-2

Note: All values are in percentage terms.

Jacobian For Citation Step Size Block:

[36] Increasing the weighting factor to 3 was sufficient to align the aggregate growth rate in the data and the model.

Table 17: Jacobian For Citation Step Size Block

Parameter	η	λ
Moment		
Public Mean - Private Mean	3	-4
Public RMS - Private RMS	3	-4

F.3 Untargeted Moments

We now discuss our model's prediction about some of the moments that we did not directly target.

Interestingly, in the data, the correlation between profitability and basic research intensity is not significantly different from zero. The same finding emerges from our model. In the baseline model, the correlation between profitability and basic research intensity is only 0.033. This result emerges because basic research investment is determined through the multi-industry presence of the firms, whereas profitability is determined by the share of hot and cold product lines, the type of research investment, and the productivity distribution $\mathcal{F}(\hat{q})$ in the economy.

Our model naturally generates a positive correlation between multi-industry presence and firm size, which is also empirically true in the data. This arises since both of these moments are strongly correlated with firm survival. In the model, we find a correlation of 0.29 between the log employment and multi-industry presence. In the data, this value is 0.76.

Another stylized fact in our data is that the firm-size distribution is highly skewed–a well-known feature that is documented extensively in the literature. For detailed references, see Aghion, Akcigit, and Howitt (2014). In our model, we capture this fact with a skewness of the firm-size distribution of 3.47. This value is 3.07 in the data.

Our estimates indicate that entrants play an important direct role in overall growth. The growth rate contribution of entrants is 0.43%, whereas that number is 0.86% for incumbents. This implies that entrants account for 32% of growth. Though our number is for the French economy, it is in line with Foster, Haltiwanger, and Krizan (2001) who find that 25% of productivity growth in the US comes from new entry.

G Quantitative Robustness Checks

In this section, we illustrate the robustness of our estimation results and policy analysis to a number of variations on our sample and model.

G.1 Heterogeneous Firm Types

Our baseline estimation identifies the degree of cross-industry spillovers associated with basic research by using information on the correlation between firms' multi-industry presence and their share of basic research spending. One concern is that this correlation could be also accounted for in an extended model in which firms have a differential comparative advantage in applied versus basic research. To explore this possibility, we formulate and re-estimate a version of the model featuring multiple, persistent firm types.

In this extended model, upon successful entry, firms randomly realize a type, denoted by k, that is fully persistent and that determines their basic research cost structure. In particular, we consider the case in which, with a certain probability $\chi \in (0,1)$, firms are of a type that does no basic research (or has an infinitely or prohibitively high fixed cost of basic research); and with the remaining probability, they are of a type that can undertake basic research but must still pay a fixed and variable cost to do so, as in the baseline model.

Formally, χ enters into the flow equations characterizing the steady-state distributions over firms Γ and products $\mathcal{F}$ (see Appendix D). The effect on incentives can be seen most directly in the modified value function below, in which a firm's production function for basic research (and, hence, overall value function) is dependent on firm type.

$$(r-g)V_m^k = \max_{a,b} \left\{ \begin{array}{c} -\tilde{w}(1-\psi_a^k)h_a(a) - \tilde{w}(1-\psi_b^k)\mathbb{E}_\phi^k \left[h_b^k(b) + \mathbf{1}_{(b>0)}\phi\right] \\ +a\left[\alpha V_H^k + (1-\alpha)V_C^k + V_m^k\right] + b(1+p\frac{m}{M})\left[V_H^k + V_m^k\right] \\ +x\left(1-\frac{m}{M}\right)\left[V_{m+1}^k - V_m^k\right] - \tau V_m^k + \kappa \mathbb{E}_{\hat{q}} V(\hat{q}) \end{array} \right\}$$

It is worth noting that adding firm types creates a substantially more complex problem due to the need to track and simulate the joint distribution of firm type and industry presence, as well as productivity.

To identify the parameter χ, we use information from the subsample of firms that appear multiple times in the R&D survey. Specifically, we match the overall fraction of firms performing basic research, as well as the persistence of the performance of basic research from year to year. In this sample, the unconditional probability of doing basic research is 30%, while the persistence is 74%. Intuitively, the more that basic research is driven by fixed firm types, rather than by ephemeral, idiosyncratic draws from a fixed cost distribution, the more persistent basic research should be. Thus, the firm-type parameter and the idiosyncratic fixed-cost distribution parameters will be jointly determined by the persistence and basic research extensive margin moments. Note that we re-estimate all of the parameters of this model, not just the firm-type parameter alone. The matched moments appear in Tables 19 and 20. We match both moments relatively well: the simulated persistence is exactly matched to the empirical one at 60%, while the unconditional share of firms investing in basic research is 36%.

Table 18 checks whether the policy conclusions of the baseline model change with the inclusion of persistent types. While we see less-aggressive optimal policies across the board, the same essential message holds: a limited uniform policy can produce some welfare gains but is hampered by over-subsidization of applied research actually harming the incentives for basic research. The targeted research subsidy, which calls for a much higher basic research subsidy, can still produce substantial welfare gains. These results are largely consistent with our intuition for the implications for introducing persistent firm types. The core inefficiency present in the baseline model remains, that is, firms in a limited number of industries are not fully appropriating the fruits of their basic research investments. The fact that this is now true for only a subset of firms dampens the impact of this inefficiency, leading to lower optimal policy levels (in the uniform case, 30% vs 16%).

Interestingly, the presence of firm heterogeneity opens up the possibility for new policy instruments. In the last row of table 18, we present the case in which the policymaker can set the uniform

Table 18: Policy results for the case of persistent firm types.

Policy	ψ_a	ψ_b	R/Z	τ_a	τ_b^e	τ_u	L_p	L_b	L_u	L_e	L_a	α	g	β
Baseline	10.0	10.0	0.5	14.2	0.5	0.2	86.1	0.5	0.5	4.1	8.8	7.6	1.3	100.00
Soc Plan				11.9	1.5	2.2	85.1	2.2	2.5	3.2	6.9	26.0	1.5	101.98
Uniform	16.0	16.0	0.5	14.8	0.7	0.2	85.2	0.6	0.5	4.3	9.4	8.6	1.3	100.04
Targeted	3.8	41.7	0.5	12.5	3.5	0.2	85.4	3.9	0.5	2.7	7.4	25.6	1.4	101.48
Unif Acad	16.0	16.0	0.5	14.8	0.7	0.2	85.1	0.6	0.5	4.3	9.4	9.1	1.3	100.04
Type Dep	ψ_{nobasic}	ψ_{basic}												
Uniform	2.5	17.0	0.5	15.6	1.7	0.2	85.2	1.6	0.5	2.5	10.2	15.6	1.4	100.72

See Table 7 for variable definitions.

subsidy according to the identified firm type. The resulting optimal policy favors high-type firms and delivers welfare gains of roughly half those of the targeted policy, which has differential subsidies for applied and basic research. It is natural that we cannot achieve parity here because within high type firms, we still have the issue of oversubsidizing applied research. Yet the problem is mitigated by the presence of some (low-type) firms that are unable to do basic research, meaning that firm type is a strong proxy for research type.

Table 19: Parameters - Persistent Types

#	Description	Sym	Value	#	Description	Sym	Value
1.	Discount Rate	δ	0.039	11.	Basic Cost Curvature	ν_b	0.550
2.	CRRA Utility Parameter	γ	3.029	12.	Applied Cost Scale	ξ_a	1.286
3.	Elasticity of Subsitution	ε	6.035	13.	Basic Cost Scale	ξ_b	5.396
4.	Cross-industry Spillover	p	0.115	14.	Basic Fixed Mean	$\bar{\phi}$	4.930
5.	Basic Step Size	η	0.070	15.	Basic Fixed Std. Dev.	σ	0.342
6.	Applied Step Size	λ	0.050	16.	Product Cooldown Rate	ζ	0.119
7.	Mass of Entrants	E	0.467	17.	Buyout Rate	ι	0.457
8.	Mass of Academic Labs	U	0.514	18.	Citation Rate	x	3.028
9.	Exogenous Exit Rate	κ	0.006	19.	Basic Type Probabitity	q	0.365
10.	Applied Cost Curvature	ν_a	0.372				

Table 20: Moments - Persistent Types

Model	Data	Description	Model	Data	Description
0.2741	0.2432	Basic extensive $m = 1$	2.3036	2.2037	Mean m
0.2929	0.2308	Basic extensive $m = 2$	7.8069	6.9756	Mean m^2
0.3251	0.2703	Basic extensive $m = 3$	0.0308	0.0326	Return on sales
0.3638	0.3483	Basic extensive $m = 4$	0.0856	0.0919	Exit Rate
0.4174	0.4184	Basic extensive $m = 5$	0.0993	0.1032	Employment growth
0.4247	0.4518	Basic extensive $m = 6$	0.0125	0.0150	Aggregate growth
0.5079	0.5508	Basic extensive $m = 7$	8.2149	8.0000	Spillover differential
0.4904	0.6803	Basic extensive $m \geq 8$	0.2759	0.2603	R&D/labor
0.0532	0.0666	Basic intensive $m = 1$	12.6309	14.9965	Age, small firms
0.0585	0.0467	Basic intensive $m = 2$	21.6837	24.8733	Age, large firms
0.0667	0.0617	Basic intensive $m = 3$	8.1458	7.0130	Public Citations Mean
0.0767	0.0800	Basic intensive $m = 4$	11.8682	14.1970	Public Citations RMS
0.0902	0.0763	Basic intensive $m = 5$	5.9896	5.8850	Private Citations Mean
0.0940	0.0786	Basic intensive $m = 6$	8.8750	9.1540	Private Citations RMS
0.1150	0.1224	Basic intensive $m = 7$	0.5932	0.7372	Persistence of Basic
0.1142	0.1005	Basic intensive $m \geq 8$			

G.2 Heterogeneous Industries

To address the concern that there is heterogeneity across industries in the average intensity of basic research, we also formulate and re-estimate a version of the model with such a feature built into it. In particular, starting from the baseline of ten symmetric industries, we split the economy into two groups of industries. Each group comprises five industries that are differentiated by their parameters governing basic research costs and benefits. In the implementation, we allow the spillover probability and the basic research fixed and variable cost parameters to vary between these two groups. We then estimate these parameters using moments similar to those from the baseline estimation but split by sector.

To accommodate this extension, the necessary changes to the model and the estimation are quite substantial. The value function relevant state is now no longer just the number of industries in which the firm has working knowledge, m, but, rather, the respective numbers for groups 1 and 2, (m_1, m_2), as firms' operations can span the two groups simultaneously. The following modified value function encapsulates this logic, wherein $k \in \{1,2\}$ now denotes the groups

$$(r-g)V^k_{m_1,m_2} = \max_{a,b} \left\{ \begin{array}{c} -\tilde{w}(1-\psi^k_a)h_a(a) - \tilde{w}(1-\psi^k_b)\mathbb{E}^k_\phi\left[h^k_b(b) + \mathbf{1}_{(b>0)}\phi\right] \\ +a\left[\alpha V^k_H + (1-\alpha)V^k_C + V^k_{m_1,m_2}\right] \\ +b(1+p\frac{m_k}{M})\left[V^k_H + V^k_{m_1,m_2}\right] + p\frac{m_{-k}}{M}\left[V^{-k}_H + V^{-k}_{m_1,m_2}\right] \\ +x^1\left(1-\frac{m_1}{M_1}\right)\left[V^k_{m_1+1,m_2} - V^k_{m_1,m_2}\right] \\ +x^2\left(1-\frac{m_2}{M_2}\right)\left[V^k_{m_1,m_2+1} - V^k_{m_1,m_2}\right] \\ -\tau V^k_{m_1,m_2} + \kappa\mathbb{E}^k_{\hat{q}}V^k(\hat{q}) \end{array} \right\}$$

Notice that industry expansion in group 2 is still relevant for the value in group 1, as this affects the probability of being able to internalize spillovers from group 1 into group 2, and vice versa. Similarly, in order to compute aggregates, the distribution Γ now spans not just total industries and products, (m, n), but also those split by groups, (m_1, m_2, n_1, n_2). For our particular algorithmic parameters, this renders the computational burden larger by an order of magnitude.

To match these groups to the data, we use Figure 14 in the Appendix to group those industries with above-median average basic research intensity into a "high basic" group and the remainder in a "low basic" group. Basic research intensity in most industries lies between 3% and $8-9$%. It is highest for firms in manufacturing activities (such as "manufacture of chemicals, metals and machinery") and scientific activities ("Professional, scientific and technical activities"). The latter industry is dominated by information technology firms. These industries are also the largest contributors to basic research expenditures in aggregate terms. Basic research intensity is lowest for agriculture and finance activities. Finally, the figure suggests an outlier in terms of basic research investment, which is the industry of "Education and Human Health Activities." A closer inspection of the data reveals that most of the firms within that category are pharmaceutical/biotech companies. At the same time, their importance within the French economy seems to be rather limited in aggregate terms. The spike represents only 23 out of a total sample of 6,763 firms (representing less than 1% of aggregate basic research investment).[37]

While the industries were classified on the basis of their basic research intensity, it is interesting to note that the sensitivity of basic research to multi-industry presence changes. That is, the high basic group displays a strong relationship between industry presence and basic research intensity along both the extensive and intensive margins, while the low basic group shows a weaker relationship that is nearly flat. In the new estimation, some of the existing parameters naturally adjust slightly due to the new model specification. However, there are no major changes to the parameters common across groups, such as the discount rate, the CRRA parameter, or the applied cost level. As for the parameters that do vary across groups, we see a substantial difference in the rate of cross-industry spillovers ($p^h_b = 33\%$ and $p^l_b = 6\%$). Additionally, the basic research variable costs are about 20% higher, and the fixed costs are roughly 25% higher in the basic intensive group. Thus, both the

[37] In Section 3.2, we mentioned that these observations have no impact on our results.

benefits and costs are larger in the basic intensive group, but on net, they reproduce the cross-industry patterns seen in the data.

TABLE 21: POLICY RESULTS FOR THE CASE OF HETEROGENEITY ACROSS INDUSTRIES.

Policy	ψ_a	ψ_b	R/Z	τ_a	τ_b^e	τ_u	L_p	L_b	L_u	L_e	L_a	α	g	β
Baseline	10.0	10.0	0.5	12.7	0.4	0.1	86.2	0.7	0.5	5.0	15.9	7.4	0.8	100.0
Uniform	30.7	30.7	0.5	14.7	1.1	0.1	82.6	2.3	0.5	6.0	19.4	12.3	1.1	100.5
Academic	10.0	10.0	0.8	12.7	0.4	0.2	85.9	0.7	0.8	5.0	15.9	9.7	0.8	100.1
Unif Acad	30.5	30.5	0.7	14.7	1.1	0.2	82.5	2.2	0.7	6.0	19.4	13.5	1.1	100.5
Targeted	6.2	49.1	0.5	10.2	4.5	0.1	84.2	11.6	0.5	3.6	11.8	30.7	1.1	103.8
Per Group	ψ_{low}	ψ_{high}												
Uniform	26.0	34.6	0.5	14.7	1.1	0.1	82.7	2.3	0.5	6.0	19.5	12.3	1.1	100.5

Note: For the per-group policy, ψ_{high} refers to the high basic intensive industry policy, while ψ_{low} refers to the low basic intensive industry policy. See Table 7 for variable definitions.

Table 21 reports the planner's problem and associated policies. Our intuition going in was that, as with the model extension with persistent firm types, the precise implications on a per-group basis would be different (and would, in fact, be different if one were to consider sector-specific policies), but that, in the aggregate, approximate linearity of welfare and policy optima render implications similar to those seen in the baseline. This intuition was largely confirmed upon re-estimation of parameters and calculation of optimal policies. In the case of both the targeted policy and the uniform policy, the results are extremely similar to those of the baseline. Interestingly, a group-dependent uniform subsidy (reported in the last row) does not significantly improve outcomes in this economy. The full results are summarized in Tables 22 and 23.

TABLE 22: PARAMETERS - INDUSTRY HETEROGENEITY

#	Description	Sym	Value	#	Description	Sym	Value
1.	Discount Rate	δ	0.029	10.	Applied Cost Curvature	ν_a	0.352
2.	CRRA Utility Parameter	γ	3.178	11.	Basic Cost Curvature	ν_b	0.513
3.	Elasticity of Substitution	ε	6.317	12.	Applied Cost Scale	ξ_a	1.296
4.	Cross-industry Spillover 1	p_b^1	0.332	13.	Basic Cost Scale 1	ξ_b^1	6.007
4.	Cross-industry Spillover 1	p_b^2	0.055	13.	Basic Cost Scale 2	ξ_b^2	5.090
5.	Basic Step Size	η	0.064	14.	Basic Fixed Mean 1	$\bar{\phi}^1$	4.839
6.	Applied Step Size	λ	0.033	14.	Basic Fixed Mean 2	$\bar{\phi}^2$	4.563
7.	Mass of Entrants	E	0.624	15.	Basic Fixed Std. Dev.	σ	0.424
8.	Mass of Academic Labs	U	0.263	16.	Product Cooldown Rate	ζ	0.128
9.	Exogenous Exit Rate	κ	0.007	17.	Buyout Rate	ι	0.468
				18.	Citation Rate	x	2.331

Table 23: Moments - Industry Heterogeneity

Model	Data	Description
0.1677	0.2379	1 Basic Extensive $m = 1$
0.2048	0.2175	1 Basic Extensive $m = 2$
0.2453	0.2256	1 Basic Extensive $m = 3$
0.2818	0.2969	1 Basic Extensive $m = 4$
0.3144	0.2751	1 Basic Extensive $m = 5$
0.3495	0.2901	1 Basic Extensive $m = 6$
0.3668	0.3162	1 Basic Extensive $m = 7$
0.3458	0.3119	1 Basic Extensive $m \geq 8$
0.0279	0.0686	1 Basic Intensive $m = 1$
0.0363	0.0406	1 Basic Intensive $m = 2$
0.0461	0.0646	1 Basic Intensive $m = 3$
0.0555	0.0729	1 Basic Intensive $m = 4$
0.0641	0.0546	1 Basic Intensive $m = 5$
0.0737	0.0905	1 Basic Intensive $m = 6$
0.0791	0.0905	1 Basic Intensive $m = 7$
0.0724	0.0946	1 Basic Intensive $m \geq 8$
0.1792	0.2519	2 Basic Extensive $m = 1$
0.1901	0.2208	2 Basic Extensive $m = 2$
0.1995	0.2509	2 Basic Extensive $m = 3$
0.2079	0.2519	2 Basic Extensive $m = 4$
0.2161	0.2520	2 Basic Extensive $m = 5$
0.2262	0.2303	2 Basic Extensive $m = 6$
0.2255	0.2500	2 Basic Extensive $m = 7$
0.2416	0.2329	2 Basic Extensive $m \geq 8$
0.0415	0.0717	2 Basic Intensive $m = 1$
0.0466	0.0541	2 Basic Intensive $m = 2$
0.0512	0.0629	2 Basic Intensive $m = 3$
0.0553	0.0740	2 Basic Intensive $m = 4$
0.0591	0.0735	2 Basic Intensive $m = 5$
0.0634	0.0648	2 Basic Intensive $m = 6$
0.0633	0.0541	2 Basic Intensive $m = 7$
0.0669	0.0418	2 Basic Intensive $m \geq 8$
2.2143	2.2037	Mean Industries
7.1571	6.9756	Mean Square Industries
0.0358	0.0326	Return on Sales
0.0933	0.0919	Exit Rate
0.0743	0.1032	Employment Growth
0.3323	0.2603	R&D/Labor
11.3894	14.9965	Age, Small Firms
17.6487	24.8733	Age, Large Firms
0.0083	0.0150	Aggregate Growth
7.6597	8.0000	Spillover Differential
8.1572	7.0130	Public Citations Mean
11.8880	14.1970	Public Citations RMS
4.5516	5.8850	Private Citations Mean
6.9719	9.1540	Private Citations RMS

G.3 Importance of Research Types & Institutions

In this section, we entertain various simplifications of the model in order to sharpen the intuition about the mechanisms at play. First, we eliminate basic research by both the public and private sectors and discuss the changes in the main results. To better understand which of these two factors (research types or sectors) is leading to these effects, we then remove only private basic research while maintaining public basic research.

G.3.1 Eliminating Private and Public Basic Research

Consider a model in which basic research investment by both private and public labs is eliminated. The model features only eight parameters – as opposed to the 19 seen in the baseline model – which we fully re-estimate. We report the policy results in Table 24. The estimation results are contained in Tables 25 and 26. The baseline for this economy takes as given the 10% subsidy rate.

Table 24: Policy Analysis Without Private and Public Basic Research

Policy	ψ_a	τ_a	L_p	L_e	L_a	g	β
Baseline	10.0	15.28	86.88	3.75	9.37	1.27	100.00
Uniform	-4.47	13.78	88.64	3.24	8.11	1.14	100.10

See Table 7 for variable definitions.

The first row of Table 24 reports the baseline economy, while the second row reports the optimal research subsidy. The most important result is that the optimal policy is negative, which means that the baseline economy features overinvestment in research. Qualitatively, this is the opposite of our full-fledged economy with two types of research efforts. This result has two important implications: first, as showed earlier, our model can generate over- or underinvestment depending on parameter values; second, and more important, the distinction between the two types of research and the associated spillovers turns out to be crucial for policy conclusions.

Thus, this model, in addition to being unable to make positive predictions regarding basic research, yields misleading policy conclusions. This is a byproduct of the elimination of basic research, in the form of both public and private research labs. In the next section, we reintroduce basic research by public labs in order to understand how much of the final result is due to the removal of basic research from the private sector alone.

Table 25: Parameters - Eliminating Private and Public Basic Research

#	Description	Sym	Value
1.	Discount Rate	δ	0.039
2.	CRRA Utility Parameter	γ	3.148
3.	Elasticity of Subsitution	ε	6.132
4.	Applied Step Size	λ	0.052
5.	Mass of Entrants	E	0.400
16.	Exogenous Exit Rate	κ	0.006
7.	Applied Cost Curvature	ν_a	0.398
8.	Applied Cost Scale	ξ_a	1.294
9.	Buyout Rate	ι	0.451
10.	Citation Rate	x	2.984

Table 26: Moments - Eliminating Private and Public Basic Research

Model	Data	Description
2.2612	2.2037	Mean Industries
7.4808	6.9756	Mean Square Industries
0.0328	0.0326	Return on Sales
0.0839	0.0919	Exit Rate
0.1043	0.1032	Employment Growth
0.0127	0.0150	Aggregate Growth
0.2665	0.2603	R&D/Labor
13.0568	14.9965	Age, Small Firms
21.7208	24.8733	Age, Large Firms

G.3.2 Eliminating Private Basic Research Only

We now eliminate private basic research from the model but still maintain public research labs. Thus, private investment focuses on applied research, but there are still potential spillovers from academic research. One important implication is that we can no longer use moments relative to private basic research intensity in the estimation. We allow the social planner to optimally choose a uniform research subsidy to the private sector, and additionally, we consider a joint policy over a private research subsidy and the level of funding given to public research labs. Tables 28 and 29 report estimation results, and Table 27 policy results.

Table 27: Policy Analysis Without Private Basic Research

Policy	ψ_a	ψ_b	R/Z	τ_a	τ_b^e	L_p	L_u	L_e	L_a	α	g	β
Baseline	10.0	10.0	0.50	14.4	0.13	86.5	0.5	4.2	8.8	2.14	1.24	100.00
Uniform	1.33	1.33	0.50	13.5	0.13	87.6	0.5	3.9	8.1	2.26	1.16	100.04
Unif Acad	5.41	5.41	0.87	13.9	0.43	86.7	0.9	4.0	8.3	6.91	1.23	100.19

See Table 7 for variable definitions.

The policy implications generated in Table 27 by this simplified model are again striking. First, a comparison between the baseline economy and the uniform research subsidy reveals over-investment in private applied research. The social planner increases welfare by reducing the subsidy to the private sector from 10% to a mere 1.33%. While this reduces firms' research investment and the rate of applied research, it increases resources for production and slightly increases welfare. Second, when jointly determining the research subsidy and the funding of the public sector labs, we see evidence of complementarity between private and public research policies. That is, we simultaneously see a rise in the optimal academic spending (from 0.5% to 0.87%), and an increase in the applied subsidy rate (from 1.33% to 5.41%).

Taking the models in this section together, it is evident that the existence of investment in basic research by the private and public sectors is necessary for baseline policy conclusions. Without these distinctions in research investments and their associated spillovers, optimum policy discourages research relative to the baseline economy.

Table 28: Parameters - Eliminating Private Basic Research Only

#	Description	Sym	Value
1.	Discount Rate	δ	0.039
2.	CRRA Utility Parameter	γ	3.018
3.	Elasticity of Subsitution	ε	6.213
4.	Cross-industry Spillover	p	0.117
5.	Multi-spillover Distribution	ν	0.101
6.	Basic Step Size	η	0.078
7.	Applied Step Size	λ	0.051
8.	Mass of Entrants	E	0.481
9.	Mass of Academic Labs	U	0.494
10.	Exogenous Exit Rate	κ	0.006
11.	Applied Cost Curvature	ν_a	0.361
12.	Basic Cost Curvature	ν_b	0.537
13.	Applied Cost Scale	ξ_a	1.226
14.	Basic Cost Scale	ξ_b	5.446
15.	Basic Fixed Mean	$\bar{\phi}$	4.762
16.	Basic Fixed Std. Dev.	σ	0.329
17.	Product Cooldown Rate	ζ	0.118
18.	Buyout Rate	ι	0.456
19.	Citation Rate	x	1.301

Table 29: Moments - Eliminating Private Basic Research Only

Model	Data	Description
2.2565	2.2037	Mean Industries
7.4503	6.9756	Mean Square Industries
0.0331	0.0326	Return on Sales
0.0865	0.0919	Exit Rate
0.1030	0.1032	Employment Growth
0.0124	0.0150	Aggregate Growth
8.5014	8.0000	Spillover Differential
0.2685	0.2603	R&D/Labor
12.5101	14.9965	Age, Small Firms
19.7508	24.8733	Age, Large Firms
8.4103	7.0130	Public Citations Mean
12.2424	14.1970	Public Citations RMS
5.5394	5.8850	Private Citations Mean
8.2017	9.1540	Private Citations RMS

G.4 Limited Diffusion of Private Basic Research

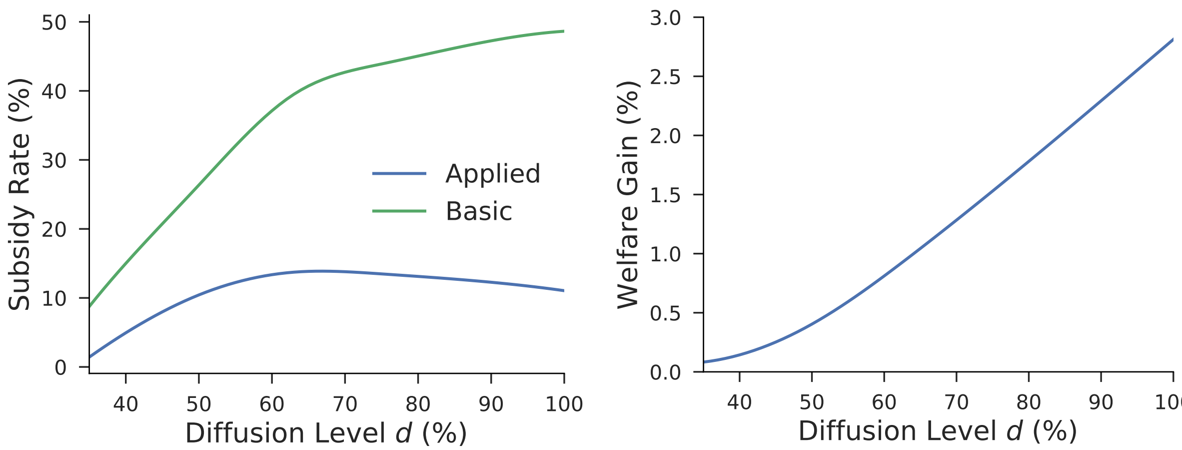
Another potential concern relates to the appropriability of public and private basic research. An important feature of research produced in the public sector is that it is a public good, while private research may be kept secret by firms. To address this concern, we start by considering a version of the model in which private basic research benefits the investing firm but makes only a limited contribution to economic growth. We parameterize this by letting only a fraction $d \in [0,1]$ of private basic research diffuse and contribute to economic growth. This changes the growth rate as follows:

$$g = \frac{\tau_a \left[\alpha \mathbb{E}^H_{\hat{q}} \left(\hat{q}+\eta\right)^{\varepsilon-1} + (1-\alpha)\mathbb{E}^C_{\hat{q}} \left(\hat{q}+\lambda\right)^{\varepsilon-1} - 1\right] + d\tau^e_b \left[\mathbb{E}_{\hat{q}} \left(\hat{q}+\eta\right)^{\varepsilon-1} - 1\right]}{\varepsilon - 1}.$$

Note that the last term in the numerator is multiplied by d, which captures the fact that only a fraction of private basic research contributes to growth. A second implication of this modification is that the equilibrium fraction of hot product lines is also reduced.

The policy implications of the new model are illustrated in Figure 15. Note that our baseline model lies at the right-hand side of the graphs, where the level of diffusion is maximal. As the fraction of diffused private basic research declines, the optimal policy provides less support to private basic research. Since a smaller fraction of private basic research is spilling over to applied research effectiveness, the optimal policy also reduces its support to applied research.

Figure 15: Optimal Policies & Welfare with Limited Diffusion of Private Basic Research



Hence, while the qualitative implications of the optimal policy remain the same – i.e., relatively more support for basic research – the magnitudes of the subsidy rates are lower. Nonetheless, for diffusion levels above roughly 60%, the optimal policy results are relatively similar to the baseline.

G.5 Applied Research Spillovers

A final question is whether applied research is necessarily targeted to specific product lines/industries. In other words, within a Schumpeterian framework, applied research might also generate multiple cross-industry spillovers. We address this concern both theoretically and empirically.

First, we generalize the structure of the spillovers by allowing for the possibility of applied cross-industry spillovers. Thus, instead of having one parameter p that specifies the probability of a basic spillover occurring, we have now parameters p_a and p_b that specify that probability for applied and basic research, respectively. The following equation defines the newly augmented value function of a firm

$$(r-g)V_m = \max_{a,b}\left\{\begin{array}{c} -\tilde{w}(1-\psi_a)h_a(a) - \tilde{w}(1-\psi_b)\mathbb{E}_\phi\left[h_b(b) + \mathbf{1}_{(b>0)}\phi\right] \\ +a(1+p_a\frac{m}{M})\left[\alpha V_H + (1-\alpha)V_C + V_m\right] + b(1+p_b\frac{m}{M})\left[V_H + V_m\right] \\ +x\left(1-\frac{m}{M}\right)\left[V_{m+1} - V_m\right] - \tau V_m + \kappa\mathbb{E}_{\hat{q}}V(\hat{q}) \end{array}\right\}$$

Empirically, we estimate the correlation between the applied research investment of firms and their multi-industry presence. To disentangle the relative research incentives in basic and applied research from their level effect, we compute the amount of each research investment relative to total sales. As with our previous benchmark specification, we also control for firm size. Table 30 reports the results.

Table 30: Basic and Applied Research to Sales

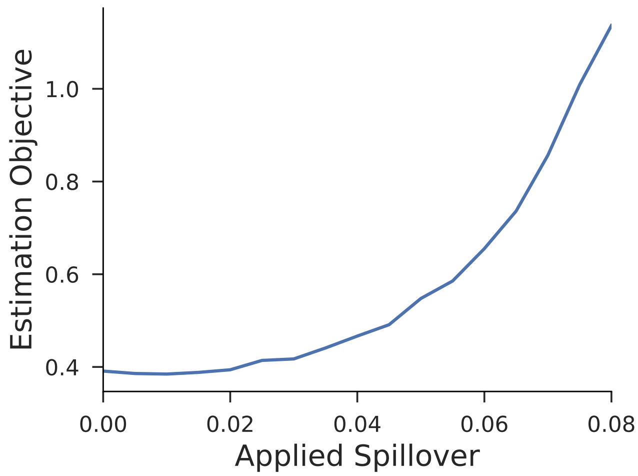
	Applied/Sales (1)	Basic/Sales (2)
Log # of 1 Digit Product Markets	0.0008	0.0024***
	(0.0025)	(0.0004)
Log Employment	-0.0120***	-0.0001
	(0.0007)	(0.0001)
Year & Organization FE	Yes	Yes
N	13708	13708

Notes: Pooled data for the period 2000-2006. Estimates are obtained using Tobit models and relate to the marginal effect of the variables at the sample mean. *Basic/Sales* is defined as the ratio of total firm investment in basic research divided by total firm sales. *Applied/Sales* is defined as the ratio of total firm investment in applied research divided by total firm sales. *Log # of Industries* is the number of distinct SIC codes in which a firm is present. *Log Employment* is the total employment of firms. *Year FE* denotes year fixed effects, and *Organization FE* denotes whether the firm operates its activity as a conglomerate or as a business group. See the Appendix for the definition of variables. Robust standard errors clustered at the firm level are in parentheses. One star denotes significance at the 10% level; two stars denote significance at the 5% level; and three stars denote significance at the 1% level.

In column 1, the dependent variable is applied research to total sales, and the correlation with firm scope is economically and statistically close to 0. Note that this is not an artifact of the variable definition since basic research investment to sales in column 2 is increasing with the multi-industry presence of a firm.

In addition to our reduced-form analysis, we quantitatively assess how the introduction of the applied spillover would affect the model fit. To this end, we evaluate the estimation objective function for different positive values of the applied spillover parameter, while keeping the other parameters fixed. The results of this exercise are plotted in Figure 16.

Figure 16: Loss Function for Values of Applied Spillover p_a



As is apparent from the graph, the objective function (wherein lower is better) is sharply increasing from the lower bound of $p_a = 0$. There is a small region around $p_a \in [0, 2\%]$ where the objective might be interpreted as flat. We also verified quantitatively that none of the values in that region has any impact on our policy results.

G.6 External CRRA Parameter

In the benchmark estimation, the CRRA parameter (γ) in the utility function is sensitive to data on firm growth. While the resulting estimate for risk aversion of 3.0 is consistent with estimates from the asset pricing literature (Gormsen and Jensen, 2017), we check the robustness of our conclusion by bringing in other estimates from the literature. Barro (2006) finds that a parameter value between 2 and 4 is necessary to match data on both overall savings rates and the change in the savings rate with income. Thus, our value lies in the middle of this range. To investigate the robustness of our estimated value, we also consider the case of a CRRA parameter equal to 2. In this exercise, we fix the value of the CRRA parameter to 2 and re-estimate in Tables 32 and 33 all other parameters to match the same moments as in the baseline estimation. Table 31 shows that all of our policy implications remain robust.

TABLE 31: POLICY ANALYSIS IN CASE OF $\gamma = 2$

Policy	ψ_a	ψ_b	R/Z	τ_a	τ_b^e	τ_u	L_p	L_b	L_u	L_e	L_a	α	g	β
Baseline	10.0	10.0	0.5	15.1	0.6	0.1	84.7	0.6	0.5	4.7	9.5	6.8	1.4	100.0
Uniform	41.3	41.3	0.5	19.2	2.0	0.0	77.8	2.1	0.5	6.5	13.1	15.6	2.0	102.2
Targeted	10.5	46.3	0.5	13.2	4.3	0.1	82.7	5.0	0.5	3.9	7.9	28.8	1.7	104.0
Unif Acad	42.9	42.9	0.8	19.5	2.0	0.3	77.1	2.1	0.8	6.6	13.4	19.5	2.1	102.4

See Table 7 for variable definitions.

TABLE 32: PARAMETERS - $\sigma = 2$

#	Description	Sym	Value
1.	Discount Rate	δ	0.039
2.	CRRA Utility Parameter	γ	2.000
3.	Elasticity of Subsitution	ε	5.744
4.	Cross-industry Spillover	p	0.116
5.	Multi-spillover Distribution	ν	0.102
6.	Basic Step Size	η	0.079
7.	Applied Step Size	λ	0.050
8.	Mass of Entrants	E	0.499
9.	Mass of Academic Labs	U	0.497
10.	Exogenous Exit Rate	κ	0.006
11.	Applied Cost Curvature	ν_a	0.358
12.	Basic Cost Curvature	ν_b	0.538
13.	Applied Cost Scale	ξ_a	1.234
14.	Basic Cost Scale	ξ_b	5.459
15.	Basic Fixed Mean	$\bar{\phi}$	4.721
16.	Basic Fixed Std. Dev.	σ	0.327
17.	Product Cooldown Rate	ζ	0.118
18.	Buyout Rate	ι	0.464
19.	Citation Rate	x	2.972

Table 33: Moments - $\sigma = 2$

Model	Data	Description
0.2475	0.2432	Basic Extensive $m = 1$
0.2830	0.2308	Basic Extensive $m = 2$
0.3196	0.2703	Basic Extensive $m = 3$
0.3570	0.3483	Basic Extensive $m = 4$
0.3946	0.4184	Basic Extensive $m = 5$
0.4322	0.4518	Basic Extensive $m = 6$
0.4695	0.5508	Basic Extensive $m = 7$
0.5181	0.6803	Basic Extensive $m >= 8$
0.0446	0.0666	Basic Intensive $m = 1$
0.0527	0.0467	Basic Intensive $m = 2$
0.0613	0.0617	Basic Intensive $m = 3$
0.0705	0.0800	Basic Intensive $m = 4$
0.0802	0.0763	Basic Intensive $m = 5$
0.0903	0.0786	Basic Intensive $m = 6$
0.1007	0.1224	Basic Intensive $m = 7$
0.1149	0.1005	Basic Intensive $m >= 8$
2.2967	2.2037	Mean Industries
7.7515	6.9756	Mean Square Industries
0.0328	0.0326	Return on Sales
0.0949	0.0919	Exit Rate
0.1025	0.1032	Employment Growth
0.0140	0.0150	Aggregate Growth
8.2431	8.0000	Spillover Differential
0.2800	0.2603	R&D/Labor
11.4272	14.9965	Age, Small Firms
18.6032	24.8733	Age, Large Firms
8.6165	7.0130	Public Citations Mean
12.5342	14.1970	Public Citations RMS
5.8124	5.8850	Private Citations Mean
8.6591	9.1540	Private Citations RMS