Introduction

- In theory, GPUs are very fast, $\sim 1e10^9$ FLOPS
- Not all problems are suited to these devices
- The trick is knowing when it's worth it

Architecture

- One GPU typically consists of hundreds of cores
- Each core executes one **thread**, cores are dynamically clustered into **blocks**
- User writes a function (called a **kernel**) that executes concurrently on many cores
- Device usually has $\sim 1 - 2$ GB of global memory
- Blocks have fast shared memory banks, cores have registers

GPU Diagram

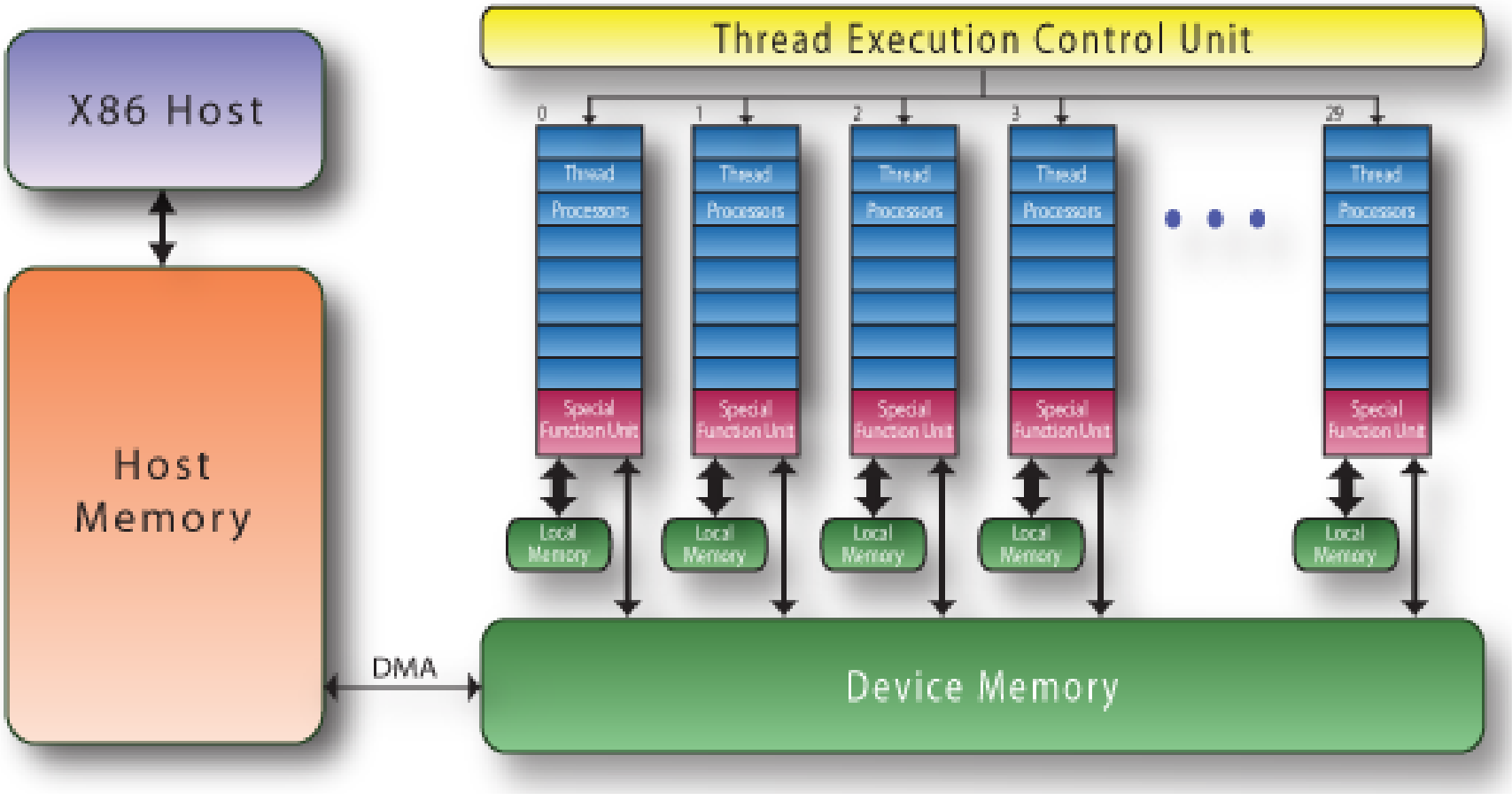


Image source: The Portland Group

Memory Access

- Threads can read/write shared and global memory, but not CPU memory
- Writes to global memory synced at end of kernel call
- Writes to shared memory can be synced within block on command
- Writes within **warp** (sets of 32 threads) are automatically synced
- Memory architecture limits amount of interdependence in problems

Interfaces

- NVIDIA's native interface is a C++-like language called CUDA
- OpenCL supports both NVIDIA and ATI - has bindings for C and Fortran
- There are interfaces with
  - MATLAB/Mathematica
  - Python/Perl
  - Fortran (PGI = $$$)

CUDA

- Compiled language based on C++
- Defines a new type of function called a kernel, to be run on GPU
- Requires basic knowledge of C and pointers
- Allows for memory allocation on GPU and calling of kernels
- Compiled using NVIDIA's `nvcc`

Simple Example I

- First we define our kernel

```
__global__ void vecAdd(double* x, double* y, int N)
{
  int i = blockIdx.x*blockDim.x + threadIdx.x;

  if (i < N) {
    y[i] = y[i] + x[i];
  }
}
```

- The variables `blockIdx.x`, `blockDim.x`, and `threadIdx.x` are given to each instance of the kernel

Simple Example II

- Now we allocate memory on the device

```
double* d_x, d_y;
int vsize = sizeof(double)*N;
cudaMalloc((void**)&d_x,vsize);
cudaMalloc((void**)&d_y,vsize);
```

- Then we copy our data over to this memory

```
cudaMemcpy(d_x,h_x,vsize,cudaMemcpyHostToDevice);
```

- Finally we execute the kernel

```
int threadsPerBlock = 16;
int numBlocks = N/threadsPerBlock; // assume divisible
vecAdd<<<numBlocks,threadsPerBlock>>>(d_x,d_y,N);
```

- Copy back to CPU using cudaMemcpy

```
cudaMemcpy(h_y,d_y,vsize,cudaMemcpyDeviceToHost);
```

Things to Note

- Global memory is not updated across threads - can't do

```
vout[i] = vin[i] + 1;
vout[i] = vout[i+1];
```

- Consecutive memory reads are coalesced within half-warps (16 threads)
- To construct 2D kernels, `threadsPerBlock` and `numBlocks` are 2D (`x` and `y`)
- Volume of blocks must not exceed 512

Multiple Functions

- Can define multiple kernels that call one another
- `__global__` keyword allows calling from CPU
- `__device__` functions can only be called from GPU

```
__device__ double utility(double c, double sigma)
{
  return (powf(c,1.0-sigma)-1.0)/(1.0-sigma);
}
```

- Device functions are automatically inlined by compiler

Shared Memory

- Shared memory - use memory across threads within block

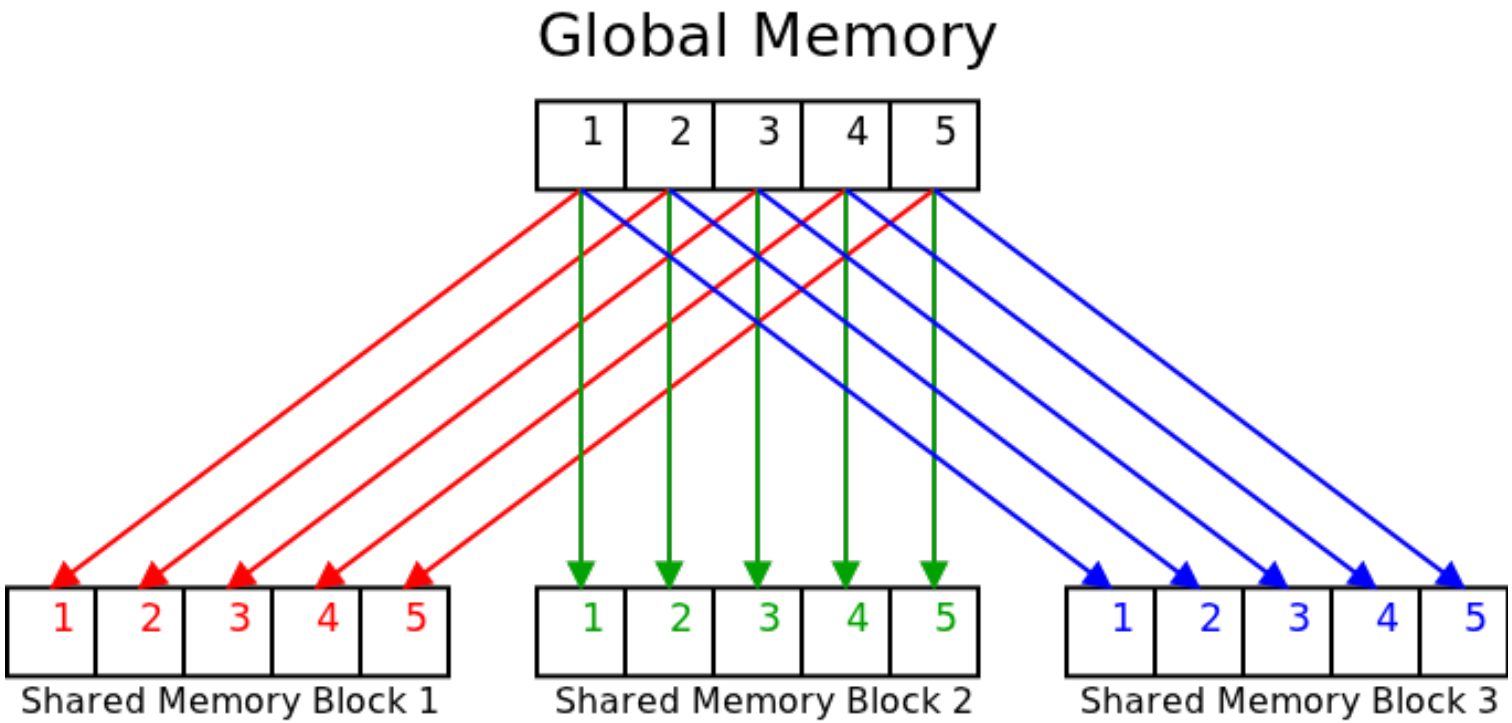
```
__global__ void vec_ma(double* vin, double* vout, double* mult)
{
  __shared__ double s_mult[M];

  int tid = threadIdx.x;
  int i = blockIdx.x*blockDim.x + tid;
  if (i < N) {
    if (tid < M) s_mult[tid] = mult[tid];
    __syncthreads();

    double v = 0.0;
    for (int j = 0; j < M; j++) {
      if (i-j >= 0) v = v + vin[i-j]*s_mult[j];
    }

    vout[i] = v;
  }
}
```

Shared Memory



Shared Memory

- Declare/allocate the memory, M must be constant

```
__shared__ double s_mult[M];
```

- Have each thread copy one value, need $M < BlockSize$

```
if (tid < M) s_mult[tid] = mult[tid];
```

- Sync data across thread block

```
__syncthreads();
```

- Data within warp is automatically synced

Value Function Iteration

- Each thread handles one state point
- Performs optimization over choice variables, updates value function
- Or you could calculate value for each state/choice pair and find max in 2D

Tricks

- Texture memory for lookup tables
- Do max absolute difference on GPU
- Utilize shared memory (transition probabilities, shock values)
- Debugging can be a pain, but there are ways to output text
- Use powers of 2 for grid size: modulus is slow, memory likes to be aligned

```
int ik = i & (nK-1);
int iz = (i-ik)/nK;
```

Reduction

- Operations like `sum` and `max` must be done differently
- Here's how to do it when `blockSize = 512`

```
int tid = threadIdx.x;
int i = blockIdx.x*blockDim.x + tid;

if (tid < N) s_data[tid] = vin[i];
__syncthreads();

if (tid < 256) s_data[tid] = fmax(s_data[tid],s_data[tid+256]); // sync
if (tid < 128) s_data[tid] = fmax(s_data[tid],s_data[tid+128]); // sync
if (tid <  64) s_data[tid] = fmax(s_data[tid],s_data[tid+ 64]); // sync
if (tid <  32) {
  s_data[tid] = fmax(s_data[tid],s_data[tid+ 32]);
  s_data[tid] = fmax(s_data[tid],s_data[tid+ 16]);
  s_data[tid] = fmax(s_data[tid],s_data[tid+  8]);
  s_data[tid] = fmax(s_data[tid],s_data[tid+  4]);
  s_data[tid] = fmax(s_data[tid],s_data[tid+  2]);
  s_data[tid] = fmax(s_data[tid],s_data[tid+  1]);
}

if (tid == 0) vout[blockIdx.x] = s_data[0];
```

Texture Memory

- Texture memory can be good for read-only memory like lookup tables
- Fast caching, hardware interpolation
- Declare texture reference

```
texture<double> texRef;
```

- Bind it to some memory

```
cudaBindTexture(0,texRef,d_tex,tex_size);
```

- Access in kernel

```
double x = 0.457;
double v = tex1D(texRef,x);
```

When To Use

- Ideal for high computation, low memory access problems
  - Many CPU problems are already memory bound
  - Not a huge speedup (< 2x) on matrix multiplication
- Local and sequential memory access patterns
  - Coalesced reads are faster
  - Global memory utilizes caching
- Avoid branching execution from conditional statements
  - Warp divergence - executes each branch serially
- You always pay the fixed cost of initial and final data transfer
  - Increasing returns

Use Cases

- Well suited to GPU
  - Value function iteration - in 1D or 2D
  - Simulation - Monte Carlo, simulated method of moments, random numbers
  - Steady states - transition matrix iteration, discrete
- Not so much
  - Sorting (e.g., bubble sort)
  - Histograms
  - VFI accelerator
- Even for the latter cases, GPU optimized algorithms are getting better

Other Interfaces

- BLAS routines come for free with CUDA runtime
- CULA Tools gives advanced linear algebra routines
- PyCUDA provides a very easy interface in Python

```
mod = SourceModule("""
  __global__ void vecAdd(double* x, double* y, int N)
  {
    int i = threadIdx.x*blockSize.x + threadIdx.x;
    if (i < N) y[i] = y[i] + x[i];
  }
  """)
func = mod.get_function("vecAdd")
func(d_x,d_y,N,block=(BS,1,1))
```

MATLAB Interface (NVIDIA only)

- Compile kernel from CUDA into PTX using `nvcc`

```
nvcc -ptx -O2 --ptxas-options=-v vecAdd.cu
```

- Load kernel into MATLAB

```
kern = parallel.gpu.CUDAKernel('vecAdd.ptx','vecAdd.cu');
kern.ThreadBlockSize = 32;
kern.GridSize = ceil(N/kern.ThreadBlockSize(1));
```

- Transfer input and allocate output

```
d_in = gpuArray(h_in);
d_out = gpuArray(N);
```

- Execute kernel and copy result back to CPU side

```
d_out = feval(kern,d_out,d_in,N);
h_out = gather(d_out);
```

- Can also write MEX file in CUDA

Hardware

- You may need to buy a new graphics card
- Can run from \$100-\$300+ depending on what you want
- Rapidly changing landscape - convergence of GPUs and CPUs?

References

- NVIDIA
  `developer.nvidia.com/category/zone/cuda-zone`
- CUDA Tookit (nvcc)
  `developer.nvidia.com/cuda-toolkit-40`
- Jesus's paper on applications to VFI and associated code
- PyCUDA `mathema.tician.de/software/pycuda`